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Pharmacy
17/MHS11/054
Mat 104

1) $Y = \frac{1}{x-2}$

from the function to be undefined the denominator must be $= 0$

$x - 2 = 0$

$x = 2$

The function $y = \frac{1}{x-2}$ is (defined) for all real numbers except 2.

Domain Real numbers except 2

Co domain - Real numbers

2 if $K = \ln v$; differentiate K

$\frac{dK}{dv} = \frac{1}{v}$

3 Express Y as an explicit function of x in the following

a) $2x - 3y - 2 = 0$

$2x - 3y = 2$

$\frac{-3y}{-3} = \frac{2 - 2x}{-3}$

$y = \frac{-2 + 2x}{3}$

b: $x^2 + y^2 = 4$

$y^2 = 4 - x^2$

$y = \sqrt{4 - x^2}$

$y = \pm\sqrt{4 - x^2}$

4. $y = \sin^{-1}$

$\Rightarrow y = \frac{x}{\sin}$

$x = \sin y$

differentiating both sides with y

$\frac{dx}{dy} = \cos y$

$\frac{dy}{dx} = \frac{1}{\cos y}$

Recall

$\cos^2 y + \sin^2 = 1$

$\cos^2 y = 1 - \sin^2 y$

$\cos y = \sqrt{1 - \sin^2 y}$

$\cos y = \sqrt{1 - x^2}$ since $\sin y = x$

hence

$\frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-x^2}}$

$\therefore$

$\frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-t^2}}$

5. If $f(x) = 2x^2 - 5$ and $g(x) = 4x - 2$, find $f \circ g(x)$ and $g \circ f(x)$

if $f(x) = 2x^2 - 5$

$g(x) = 4x - 2$

a) $f \circ g(x)$

b) $g \circ f(x)$

solution

a) $(f \circ g)x$

$f = (g(x))$

$f(x) = 2x^2 - 5$

$g(x) = 4x - 2$

$(f \circ g)x = f(g(x))$

$f(4x-2)$

$2(4x-2)^2 - 5$

$2(4x-2)(4x-2) - 5$

$2(16x^2 + 4 - 8x - 8x) - 5$

$2(16x^2 + 4 - 8x - 8x) - 5$

$\cancel{2(16} \; 32x^2 - 32x + 8 - 5$

$32x^2 - 32x + 3$ //

b $(g \circ f)x$

$g(f(x))$

$f(x) = 2x^2 - 5$

$g(x) = 4x - 2$

$g(2x^2 - 5)$

$4(2x^2 - 5) - 2$

$8x^2 - 20 - 2$

$8x^2 - 22$ //

6 $f_e(x) = \dfrac{f(x) + f(-x)}{2}$

$f(-x) = 3(-x)^2 - 2(-x) + 1$

$= 3x^2 + 2x + 1$

$= \dfrac{3x^2 - 2x + 1 + 3x^2 + 2x + 1}{2}$

$= \dfrac{3x^2 + 3x^2 - \cancel{2x} + \cancel{2x} + 2}{2}$

$= \dfrac{6x^2 + 2}{2}, \; \dfrac{\cancel{2}(3x^2 + 1)}{\cancel{2}}$

$= 3x^2 + 1$

$f_o = f(x) - f(-x)$

$$= \frac{(3x^2 - 2x + 1) - (3x^2 + 2x + 1)}{2}$$

$$= 3x^2 - 2x + 1 - 3x^2 + 2x + 1$$

$$\frac{3x^2 - 2x + 1 - 3x^2 - 2x -}{2}$$

$$= \frac{3x^2 - 3x^2 - 2x - 2x - 1 + 1}{2}$$

$$= \frac{-4x}{2} = -2x$$

$F(x) = f_e(x) + f_o(x)$

$f(x) = 3x^2 + 1 - 2x$

$= 3x^2 - 2x + 1$ //

7 Differentiate $y = \cos^2$ from first principles

Soln

$y = \cos x$

$y + \delta y = \cos(x + \delta x)$

Subtract y from both sides

$\delta y = \cos(x + \delta x) - y$

$y = \cos x$

$\delta y = \cos(x + \delta x) - \cos x \quad \ldots (1)$

Consider from trig

$(\cos A + B) = \cos A \cos B - \sin A \sin B$

$(\cos A - B) = \cos A \cos B + \sin A \sin B$

$(\cos A + B) - (\cos A - B)$

$= -2 \sin A \sin B \quad \ldots (2)$

Compare equ 1 and 2

Let

$A + B = x + \delta x \quad \ldots (i)$

$A - B = x \quad \ldots (ii)$

Adding (i) and (ii)

$2A = 2x + \delta x$

$A = \frac{2x + \delta x}{2}$

$A = x + \frac{\delta x}{2}$ } 3+

$B = \frac{\delta x}{2}$

Comlare A and 2A

$\cos(x + \delta x) - \cos x = -2\sin\left(x + \frac{\delta x}{2}\right)\sin\left(\frac{\delta x}{2}\right)$

$\delta y = -2\sin\left(x + \frac{\delta x}{2}\right)\sin\left(\frac{\delta x}{2}\right)$

$$\frac{\delta y}{\delta x} = \frac{-2\sin\left(x + \frac{\delta x}{2}\right)\cdot\sin\left(\frac{\delta x}{2}\right)}{\delta x}$$

$$\frac{\delta y}{\delta x} = \frac{-\sin\left(x + \frac{\delta x}{2}\right)\sin\left(\frac{\delta x}{2}\right)}{\delta x/2}$$

To find Std Limit

$$\lim \frac{\sin\left(\frac{\delta x}{2}\right)}{\frac{\delta x}{2}} = 1$$

$\lim -\sin\left(x + \frac{\delta x}{2}\right) \sin\frac{\delta x}{2} / \delta x/2$

$= -\sin(x + 0)\cdot 1$

8 find $\frac{dy}{dx}$ if $y = 3t^2$ and $x = 1/t^2$

if $y = 3t^3$

$x = 1/t^2$

find dy/dx

$y = 3t^2$, $\quad \frac{dy}{dt} = 6t$

$x = \frac{1}{t^2}$, $\quad \frac{dx}{dt} = -2t^{-3} = \frac{-2}{t^3}$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{6t}{\frac{-2}{t^3}} = -6t \times \frac{t^3}{-2}$$

$$= 3t^{-4}$$

9 dy/dx if $y = x^2 \cos 2x e^{4x}$

Soln

$\ln y = \ln(x^2 \cos 2x e^{4x})$

$\ln y = \ln x^2 + \ln \cos 2x + \ln e^{4x}$

$\frac{d}{dx}(\ln y) = \frac{d}{dx}(\ln x^2) + \frac{d}{dx}(\ln \cos 2x) + \frac{d}{dx}(\ln e^{4x})$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x^2}(2x) + \frac{1}{\cos 2x}(-2\sin 2x) + \frac{1}{e^{4x}}(4e^{4x})$

$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x} - \frac{2\sin 2x}{\cos 2x} + 4$

$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x} - 2\tan 2x + 4$

multiplying both sides by y,

$\frac{dy}{dx} = y\left(\frac{2}{x} - 2\tan \tan 2x + 4\right)$

but $y = x^2 \cos 2x e^{4x}$

$$\frac{dy}{dx} = x^2 \cos 2x e^{4x}\left(\frac{2}{x} - 2\tan 2x + 4\right)$$

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1b) $Y = \sin(3x^3+5)$

let $u = 3x^3+5$

$Y = \sin u$

$\frac{du}{dx} = 9x^2$

$\frac{dy}{du} = \cos u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= \cos u \times 9x^2$

$= 9x^2 \cos u$

but $u = 3x^3+5$

$\frac{dy}{dx} = 9x^2 \cos(3x^3+5)$.

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