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Examine whether or not these pair of lines are perpendicular to each other.

(1) $y-3x-2=0$ and $3y+x+9=0$

(2) $5y-4=2x+3$ and $y-5=x+6$

(3) Find the equations of the tangent and normal to the curve $x^2+y^2+3xy-11=0$ at point $x=1, y=2$.

Sol

1. $y-3x-2=0 \cdots (i)$

$3y+x+9=0 \cdots (ii)$

$M_1 M_2 = -1$

From equ(i)

$y-3x-2=0$

$y=3x+2$

$y=mx+c$

therefore $M_1=3$

From equ(ii)

$3y+x+9=0$

$y=-\frac{x}{3}-\frac{9}{3}$

$y=-\frac{x}{3}-3$

$y=mx+c$

$\therefore M_2=-1/3$

$M_1 M_2=-1$

$M_1 M_2 = 3 \times -\frac{1}{3} = -1$

Since $M_1 M_2 = -1$, therefore $y-3x-2=0$ and $3y+x+9=0$ are perpendicular

2. $3y-4=2x+3 \cdots (i)$

$y-5=x+6 \cdots (ii)$

$M_1 M_2 = -1$

From equ(i)

$3y-4=2x+3$

$3y = 2x + 3 + 4$

$y = \frac{2x}{3} + \frac{3}{3} + \frac{4}{3}$

$y = \frac{2x}{3} + \frac{7}{3}$

$y = mx + c$

$M_1 = \frac{2}{3}$

From equation

$y - 5 = x + 6$

$y = x + 6 + 5$

$y = x + 11$

$y = mx + c$

$M_2 = 1$

$M_1 \cdot M_2 = -1$

$M_1 \cdot M_2 = \frac{2}{3} \times 1 = \frac{2}{3}$

$M_1 \cdot M_2$ is not equal to -1 i.e $M_1 \cdot M_2 \neq -1$

$\therefore 3y - 4 = 2x + 3$ and $y - 5 = x + 6$ are **not** perpendicular

3 $x^2 + y^2 + 3xy - 11 = 0 \quad (^{x}_{1}, ^{y}_{2})$

$\frac{dy}{dx} = 2x + 2y\frac{dy}{dx} + 3\left(x \cdot \frac{dy}{dx} + y \cdot 1\right) = 0$

$2x + 2y\frac{dy}{dx} + 3\left(x \cdot \frac{dy}{dx} + y\right) = 0$

$2x + 2y\frac{dy}{dx} + 3x\frac{dy}{dx} + 3y = 0$

$2y\frac{dy}{dx} + 3x\frac{dy}{dx} = -2x - 3y$

$$\frac{dy}{dx}(2y+3x) = -2x-3y$$

$$\frac{dy}{dx} = \frac{-2x-3y}{2y+3x}$$

$$\frac{dy}{dx}\Big|_{x=1,y=2} = \frac{-2(1)-3(2)}{2(2)+3(1)} = -\frac{8}{7}$$

$$y-y_1 = m(x-x_1)$$

$$y-2 = \frac{-8}{7}(x-1)$$

$$7y-14 = -8x-8$$

$$7y+8x-14-8=0$$

$7y+8x-22=0 \Rightarrow$ Equation of tangent

1 $y-y_1 = M_2(x-x_1)$

$M_1 \cdot M_2 = -1$

$\frac{-8}{7} \cdot M_2 = -1$

$M_2 = \frac{-1}{\frac{-8}{7}}$

$M_2 = \frac{7}{8}$

$$y-2 = \frac{7}{8}(x-1)$$

$$8(y-2) = 7(x-1)$$

$$8y-16 = 7x-7$$

$$8y-7x-16+7=0$$

$8y-7x-9=0 \Rightarrow$ Equation of Normal