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Examine whether or not these pair of lines are perpendicular to each other:

1. $y - 3x - 2 = 0$ and $3y + x + 9 = 0$
2. $3y - 4 = 2x + 3$ and $y - 5 = x + 6$
3. Find the equations of the tangent and normal to the curve ~~x equation~~ $x^2 + y^2 + 3xy - 11 = 0$ at the point $x = 1, y = 2$

Solution

1. $y - 3x - 2 = 0$

$3y + x + 9 = 0$

For the lines to be perpendicular then $M_1 M_2 = -1$

$y - 3x - 2 = 0$

making y the subject of the formula

$y = 3x + 2$

Comparing with $y = mx + c$.

$M_1 = 3$

$3y + x + 9 = 0$

making y the subject of the formula

$3y = -x - 9$

$y = \frac{-x}{3} - \frac{9}{3}$

$y = -\frac{x}{3} - 3$

$y = mx + c \quad . \quad M_2 = -1/3$

$M_1 M_2 = -1$ for perpendicularity

$3 \times -1/3 = -1$ since $M_1 M_2 = -1$

Then the lines $y - 3x - 2 = 0$ and $3y + x + 9 = 0$ Are Perpendicular

2 $3y - 4 = 2x + 3$ . . ①

$y - 5 = x + 6$ . . ②

$3y - 4 = 2x + 3$

making y the subject of the formula

$3y = 2x + 3 + 4$

$3y = 2x + 7$

$y = \frac{2x}{3} + \frac{7}{3}$

By comparing $y = mx + c$

$m_1 = 2/3$

$y - 5 = x + 6$

making y the subject of the formula

$y - 5 = x + 6$

$y = x + 6 + 5$

$y = x + 11$

By comparing with $y = mx + c$

$m_2 = 1$

But for the lines to be perpendicular,

$m_1 m_2 = -1$

$m_1 m_2 = 2/3 \times 1 = 2/3$

$m_1 m_2 \neq 1$

Hence, the lines $3y - 4 = 2x + 3$ and $y - 5 = x + 6$ are not Perpendicular

3 $x^2 + y^2 + 3xy - 11 = 0 \quad (x = 1, y = 2)$

$m = \frac{dy}{dx}$

$x^2 + y^2 + 3xy - 11 = 0$

$\frac{dy}{dx}$

$2x + 2y\frac{dy}{dx} + 3\left(x \cdot \frac{dy}{dx} + y \cdot 1\right) = 0$

$2x + 2y\frac{dy}{dx} + 3\left(x \cdot \frac{dy}{dx} + y\right) = 0$

$2x + \frac{2y\,dy}{dx} + \frac{3x\,dy}{dx} + 3y = 0$

$\frac{2y\,dy}{dx} + \frac{3x\,dy}{dx} = -2x - 3y$

$\frac{dy}{dx}(2y + 3x) = -2x - 3y$

$\frac{dy}{dx} = \frac{-2x - 3y}{2y + 3x}$

$m = \frac{dy}{dx}\Big|_{x=1, y=2} = \frac{-2(1) - 3(2)}{2(2) + 3(1)} = \frac{-2 - 6}{4 + 3} = \frac{-8}{7}$

a Equation of tangent

$y - y_1 = m(x - x_1)$

$y - 2 = -\frac{8}{7}(x - 1)$

$7(y - 2) = -8(x - 1)$

$7y - 14 = -8x + 8$

$7y + 8x = 8 + 14$

$7y + 8x = 22$

$7y + 8x - 22 = 0$ is the equation of tangent.

b Equation of Normal.

$$y - y_1 = -1/m\,(x - x_1)$$

$$y - 2 = -1/(-8/7)\,(x - 1)$$

$$y - 2 = \frac{7}{-8}(x - 1)$$

$$-8(y - 2) = 7(x - 1)$$

$$-8y - 16 = 7x - 7$$

$$-8y - 7x = -7 + 16$$

$$-8y - 7x = 9$$

$-8y - 7x - 9 = 0$ is the equation of normal.