

③ $x^2 + y^2 + 3y - 11 = 0$ at point $C(1, 2)$
 $2x + 2y \frac{dy}{dx} + 3 \left(x + \frac{dy}{dx} \right) - 0 = 0$

$$2x + 2y \frac{dy}{dx} + 3x \frac{dy}{dx} + 3 = 0$$

$$2y \frac{dy}{dx} + 3x \frac{dy}{dx} = -2x - 3$$

$$\frac{dy}{dx} = \frac{-2x - 3}{2y + 3x}$$

$$m = \frac{dy}{dx} = \frac{-(2x + 3y)}{2y + 3x}$$

When $x=1$ and $y=2$

$$m = \frac{-[2(1) + 3(2)]}{2(2) + 3(1)}$$

$$= \frac{-(2+6)}{4+3} = \frac{-8}{7}$$

$$m = \frac{-8}{7}$$

Equation of the tangent to a curve

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{8}{7}(x - 1)$$

$$y - 2 = -\frac{8x}{7} + \frac{8}{7}$$

$$7y - 14 = -8x + 8$$

$$8x + 7y - 14 - 8 = 0$$

$$8x + 7y - 22 = 0$$

b. Equation of the normal to a curve

$$y - y_1 = -\frac{1}{m}(x - x_1)$$

$$y - 2 = -\frac{1}{-\frac{8}{7}}(x - 1)$$

$$y - 2 = \frac{7}{8}(x - 1)$$

$$y - 2 = \frac{7x}{8} - \frac{7}{8}$$

$$8y - 16 = 7x - 7$$

$$8y = 7x - 7 + 16$$

$$7x - 8y + 9 = 0 \quad \parallel$$

1) $y - 3x - 2 = 0$ and $3y + x + 9 = 0$

Let $A = y - 3x - 2 = 0$

$$= \frac{dy}{dx} - 3 - 0 = 0$$

$$= \frac{dy}{dx} - 3 = 0$$

$$= \frac{dy}{dx} = 3$$

Let $B = 3y + x + 9 = 0$

$$3 \frac{dy}{dx} + 1 + 0 = 0$$

$$3 \frac{dy}{dx} + 1 = 0$$

$$\frac{dy}{dx} = -\frac{1}{3}$$

$$A \perp B$$

i.e. $y - 3x - 2 = 0$ is perpendicular to $3y + x + 9 = 0$

2) $3y - 4 = 2x + 3$ and $y - 5 = x + 6$

Let $A = 3y - 4 = 2x + 3$

$$3 \frac{dy}{dx} - 0 = 2 + 0$$

$$3 \frac{dy}{dx} = 2$$

$$\frac{dy}{dx} = \frac{2}{3}$$

Let $B = y - 5 = x + 6$

$$\frac{dy}{dx} - 0 = 1 + 0$$

$$\frac{dy}{dx} = 1$$

$$\therefore A \not\perp B$$

i.e. $3y - 4 = 2x + 3$ and $y - 5 = x + 6$ is not perpendicular.