

Name: Ibeneme Chioma Sarah

Dept: MBBS

College: MHS

Matric no: 19/MHS01/188

Serial no: 070

Assignment 7

Question:

Examine whether or not these pair of lines are perpendicular to each other.

1) $y - 3x - 2 = 0$ and $3y + x + 9 = 0$

2) $3y - 4 = 2x + 3$ and $y - 5 = x + 6$

3) find the equations of the tangent and normal to the curve $x^2 + y^2 + 3xy - 11 = 0$ at the point $x = 1$ and $y = 2$

solution:

1) $y - 3x - 2 = 0 \quad \dots$
 $y = 3x + 2$

$$m_1 = \frac{dy}{dx} = 3$$

$$3y + x + 9 = 0$$

$$3y = -x - 9$$

$$y = -\frac{1}{3}x - 3$$

$$m_2 = \frac{dy}{dx} = -\frac{1}{3}$$

1 $m_1 m_2 = -1$ for two lines that are perpendicular to each other

$$m_1 m_2 = 3 \times -\frac{1}{3} = -1$$

$\therefore y - 3x - 2 = 0$ and $3y + x + 9 = 0$ are perpendicular

to each other.

$$2) 3y - 4 = 2x + 3$$

$$3y = 2x + 3 + 4 = \frac{(+2)}{3}x - \frac{(-1)}{3} \Rightarrow y = \frac{2}{3}x + \frac{7}{3}$$

$$y = \frac{2}{3}x + \frac{7}{3}$$

$$m_1 = \frac{2}{3}$$

$$y - 5 = x + 6$$

$$y = x + 11$$

$$m_2 = 1$$

$m_1 m_2 = -1$ for two lines that are perpendicular to each other

$$m_1 m_2 = \frac{2}{3} \times 1 = \frac{2}{3}$$

$$m_1 m_2 \neq -1$$

$\therefore 3y - 4 = 2x + 3$ and $y - 5 = x + 6$ are not perpendicular to each other.

$$3) x^2 + y^2 + 3xy - 11 = 0$$

$$\frac{dy}{dx} = 2x + 2y \frac{dy}{dx} + 3 \left(x \frac{dy}{dx} + y \cdot 1 \right) = 0$$

$$\frac{dy}{dx} = 2x + 2y \frac{dy}{dx} + 3x \frac{dy}{dx} + 3y = 0$$

$$\frac{dy}{dx} (2 - 2y + 3x) = -2x - 3y - 11$$

$$\frac{dy}{dx} = \frac{-2x-3y}{2y+3x}$$

$$M(1,2), m = \frac{dy}{dx} = \frac{-2(1)-3(2)}{2(2)+3(1)} = \frac{-2-6}{4+3} = \frac{-8}{7}$$

Equation of the tangent:

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{8}{7}(x - 1)$$

$$7(y - 2) = -8(x - 1)$$

$$7y - 14 = -8x + 8$$

$$8x + 7y - 14 - 8 = 0$$

$$\boxed{8x + 7y - 22 = 0} \rightarrow \text{Equation of the tangent}$$

Equation of the normal:

$$y - y_1 = \frac{-1}{m_2}(x - x_1)$$

$$y - 2 = -\frac{1}{-\frac{8}{7}}(x - 1)$$

$$y - 2 = \frac{7}{8}(x - 1)$$

$$8(y - 2) = 7(x - 1)$$

$$8y - 16 = 7x - 7$$

$$7x - 8y - 7 + 16 = 0$$

$$7x - 8y + 9 = 0 \rightarrow \text{Equation of the normal}$$