Nnaemeka Okoye
Computer Engineering
19/Eng02/045
Math 104
SN: 46

1) $y = \frac{(x+1)^2(x-2)^{1/2}}{(2x-1)(x-3)^{4/3}}$

$\ln y = \ln((x+1)^2) + \ln((x-2)^{1/2}) - \ln(2x-1) - \ln((x-3)^{4/3})$

$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{(x+1)^2}\cdot 2(x+1) + \frac{1}{(x-2)^{1/2}}\cdot\frac{1}{2}(x-2)^{-1/2} - \frac{1}{2x-1}\cdot 2 - \frac{1}{(x-3)^{4/3}}\cdot\frac{4}{3}(x-3)^{1/3}$

$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{2}{x+1} + \frac{1}{2(x-2)^{1/2}(x-2)^{1/2}} - \frac{2}{2x-1} - \frac{4}{3}(x-3)^{1/3-4/3}$

$\frac{dy}{dx} = y\left[\frac{2}{x+1} + \frac{1}{2(x-2)} - \frac{2}{2x-1} - \frac{4(x-3)^{-1}}{3}\right]$

$\frac{dy}{dx} = y\left[\frac{2}{x+1} + \frac{1}{2x-4} - \frac{2}{2x-1} - \frac{4}{3x-9}\right]$

$\frac{dy}{dx} = \frac{(x+1)^2(x-2)^{1/2}}{(2x-1)(x-3)^{4/3}}\left[\frac{2}{x+1} + \frac{1}{2x-4} - \frac{2}{2x-1} - \frac{4}{3x-9}\right]$

2) $y = \frac{3e^k \sin 2k}{k^{5/2}}$

$\ln y = \ln(3e^k) + \ln(\sin 2k) - \ln(k^{5/2})$

$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{3e^k}\cdot 3e^k + \frac{1}{\sin 2k}\cdot 2\cos 2k - \frac{1}{k^{5/2}}\cdot\frac{5k^{3/2}}{2}$

$\frac{dy}{dx} = y\left[1 + \frac{2\cos 2k}{\sin 2k} - \frac{5k^{3/2-5/2}}{2}\right]$

$\frac{dy}{dx} = \frac{3e^k \sin 2k}{k^{5/2}}\left[1 + \frac{2\cos 2k}{\sin 2k} - \frac{5}{2k}\right]$

3) $\int 4\sec^2(3m+1)\,dm$

$u = 3m+1$

$\frac{du}{dm} = 3$

$du = 3dm$

$dm = \frac{du}{3}$

$= \int 4\sec^2(u) \cdot \frac{du}{3}$

$= \frac{4}{3} \int \sec^2(u)\, du$

$= \frac{4}{3} \tan(u) + C$

$= \frac{4}{3} \tan(u) + C$

$= \frac{4}{3} \tan(3m+1) + C$

4) $\int 2t(3t^2-1)^{1/2}\, dt$

$u = (3t^2-1)^{1/2}$

$u^2 = 3t^2 - 1$

$u^2 + 1 = 3t^2$

$\frac{u^2+1}{3} = t^2$

$t = \left(\frac{u^2+1}{3}\right)^{1/2}$

$\frac{dt}{du} = \frac{1}{2}\left(\frac{u^2+1}{3}\right)^{-1/2} \cdot \frac{2u}{3}$

$\frac{dt}{du} = \frac{2u}{2\times 3}\left(\frac{u^2+1}{3}\right)^{-1/2}$

$dt = \frac{u\,du}{3}\left(\frac{u^2+1}{3}\right)^{-1/2}$

$= \left(2\left(\frac{u^2+1}{3}\right)^{1/2} \cdot u \cdot \frac{u\,du}{3}\left(\frac{u^2+1}{3}\right)^{-1/2}\right.$

$= \int \frac{2u^2}{3} \cdot \left(\frac{u^2+1}{3}\right)^{1/2-1/2} du$

$= 2/3 \int u^2\, du$

$= \frac{2}{3}\left[\frac{u^3}{3}\right] + C$

$= \frac{2t^3}{9} + C$

$= \frac{2((3t^2-1)^{1/2})^3}{9} + C$

$= \frac{2(3t^2-1)^{3/2}}{9} + C$

5) $\int \frac{2x}{\sqrt{4x^2-1}}\,dx$

$u = \sqrt{4x^2-1}$

$u^2 = 4x^2 - 1$

$u^2 - 1 = 4x^2$

$\frac{u^2+1}{4} = x^2$

$x = \sqrt{\frac{u^2+1}{4}}$

$\frac{dx}{du} = \frac{1}{2}\left(\frac{u^2+1}{4}\right)^{-1/2} \cdot \frac{2u}{4}$

$\frac{dx}{du} = \frac{2u}{2\times 4}\left(\frac{u^2+1}{4}\right)^{-1/2}$

$\frac{dx}{du} = \frac{u}{4}\left(\frac{u^2+1}{4}\right)^{-1/2}$

$dx = \frac{u\,du}{4}\left(\frac{u^2+1}{4}\right)^{-1/2}$

$= \int 2\left(\frac{u^2+1}{4}\right)^{1/2} \cdot \frac{1}{u} \cdot \frac{u\,du}{4}\left(\frac{u^2+1}{4}\right)^{-1/2}$

$= \frac{1}{4}\int \left(\frac{u^2+1}{4}\right)^{1/2-1/2} du$

$= \frac{2}{4}\int du$

$= \frac{2u}{4} + C$

$= \frac{1}{2}u + C$

$= \frac{u}{2} + C$

$= \frac{\sqrt{4x^2-1}}{2} + C$