3. Find the equations of the tangent and normal to the curve $x^2 + y^2 + 3xy - 11 = 0$ at the point $x=1, y=2$

$x^2 + y^2 + 3xy - 11$ at point $(1, 2)$

$$\frac{dy}{dx} = 2x + 2y\frac{dy}{dx} + 3\left(x\frac{dy}{dx} + y - 1\right) = 0$$

$$\Rightarrow 2x + 2y\frac{dy}{dx} + 3\left(x\frac{dy}{dx} + y\right) = 0$$

$$\Rightarrow 2x + 2y\frac{dy}{dx} + 3x\frac{dy}{dx} + 3y = 0$$

$$\Rightarrow 2y\frac{dy}{dx} + 3x\frac{dy}{dx} = -2x - 3y$$

$$\frac{dy}{dx}(2y + 3x) = -2x - 3y$$

$$\frac{dy}{dx} = \frac{-2x - 3y}{2y + 3x}$$

$$\left.\frac{dy}{dx}\right|_{x=1, y=2} = \frac{-2(1) - 3(2)}{2(2) + 3(1)} = \frac{-2 - 6}{4 + 3} = \frac{-8}{7}$$

$m = -8/7$

Equation of tangent

$y - y_1 = m(x - x_1)$

$y - 2 = -8/7(x - 1)$

$7(y - 2) = -8(x - 1)$

$7y - 14 = -8x + 8$

$7y + 8x - 22 = 0$

Equation of normal

$y - y_1 = -1/m(x - x_1)$

Mathematics Assignment (MAT 104) Serial Number: 102
Name: PAPKA DIJA YUSUFU Matric Number: 19/MHS01/381
Department: MBBS (Medicine and Surgery)

Examine whether or not these pair of lines are perpendicular to each other.

1) $y - 3x - 2 = 0$ and $3y + x + 9 = 0$

$y - 3x - 2 = 0$
$3y + x + 9 = 0$

The lines are said to be perpendicular if $M_1 M_2 = -1$

$y - 3x - 2 = 0$
make y subject of formular
$\therefore y = 3x + 2$,
$y = mx + c$
$M_1 = 3$

$3y + x + 9 = 0$
make y subject of formular
$3y = -x - 9$
$y = -\frac{x}{3} - \frac{9}{3}$

$y = -\frac{x}{3} - 3$
$y = mx + c$
$\therefore M_2 = -\frac{1}{3}$

Remember $M_1 M_2 = -1$
where $M_1 = 3$, and $M_2 = -\frac{1}{3}$

$3 \times -\frac{1}{3} = -1$
$\therefore$ They are perpendicular to each other.

2) $3y - 4 = 2x + 3$, and $3y - 2x - 7$

The lines are said to be perpendicular if $M_1 M_2 = -1$

$3y - 2x - 7$
make y subject of formular
$3y = 2x + 7$
$y = \frac{2x}{3} + \frac{7}{3}$
$y = mx + c$
$M_1 = \frac{2}{3}$

2) $3y - 4 = 2x + 3$; $3y - 2x - 7 = 0$
$y - 5 = x + 6$; $y - x - 11 = 0$

The lines are said to be perpendicular if $M_1 M_2 = -1$

$3y - 2x - 7 = 0$
make y subject of formular
$3y = 2x + 7$
$y = \frac{2x}{3} + \frac{7}{3}$

$y = mx + c$
$M_1 = \frac{2}{3}$

$y - x - 11 = 0$
make y subject of formular
$y = x + 11$
$y = mx + c$
$M_2 = 1$

Recall $M_1 M_2 = -1$
$\frac{2}{3} \times 1 = \frac{2}{3}$
$\therefore$ The lines are not perpendicular to each other.

$y - y_1 = -1/8/7(x - x_1)$

$y - 2 = \frac{-7}{8}(x - 1)$

$8(y - 2) = -7(x - 1)$

$8y - 16 = -7x + 7$

$8y + 7x - 23 = 0$

$\therefore 8y + 7x - 23 = 0$