

$$① \int \frac{dx}{x^2+7}$$

$$\text{let } u = \frac{x}{\sqrt{7}} \rightarrow \frac{du}{dx} = \frac{1}{\sqrt{7}} \rightarrow dx = \sqrt{7} du$$

$$\int \frac{1}{x^2+7} = \int \frac{\sqrt{7}}{7u^2+7} du$$

$$= \frac{1}{7} \int \frac{1}{u^2+1} du$$

$$\int \frac{1}{u^2+1} du \text{ is a standard integral:}$$

$$= \arctan(u)$$

$$\frac{1}{\sqrt{7}} \int \frac{1}{u^2+1} du = \frac{\arctan(u)}{\sqrt{7}}$$

$$\text{recall } u = \frac{x}{\sqrt{7}}$$

$$\frac{\arctan(u)}{\sqrt{7}} = \frac{\arctan\left(\frac{x}{\sqrt{7}}\right)}{\sqrt{7}}$$

$$\text{Thus } \int \frac{dx}{x^2+7} = \frac{\arctan\left(\frac{x}{\sqrt{7}}\right)}{\sqrt{7}} + C$$

$$② \int \frac{dx}{x^2+64}$$

$$\text{let } u = \frac{x}{8} \rightarrow \frac{du}{dx} = \frac{1}{8} \rightarrow dx = 8 du$$

$$\int \frac{dx}{x^2+64} = \int \frac{8}{64u^2+64} du$$

$$= \frac{1}{8} \int \frac{1}{u^2+1} du$$

$$\text{Also } \int \frac{1}{u^2+1} du \text{ is standard} = \arctan(u)$$

$$\frac{1}{8} \int \frac{1}{u^2+1} du = \frac{\arctan(u)}{8}$$

$$\text{recall } u = \frac{x}{8}$$

$$\frac{\arctan(u)}{8} = \frac{\arctan\left(\frac{x}{8}\right)}{8}$$

$$\text{Thus } \int \frac{dx}{x^2+64} = \frac{\arctan\left(\frac{x}{8}\right)}{8} + C$$