Question

Examine whether or not these pair of lines are perpendicular to each other.

(1) $y - 3x - 2 = 0$ and $3y + x + 9 = 0$

Solution.

For two lines to be perpendicular to each other, $M_1 M_2 = -1$

$y - 3x - 2 = 0$

$y = 3x + 2$

by Comparing with $y = mx + c$

$m_1 = 3$.

Also, $3y + x + 9 = 0$

$3y = -x + 9$

$y = \frac{-x}{3} + 3$

$M_2 = -\frac{1}{3}$

$M_1 M_2 = 3 \times \frac{-1}{3} = -1$

Since $M_1 M_2 = -1$, the pair of lines $y - 3x - 2 = 0$ and $3y + x + 9 = 0$ are said to be perpendicular to each other.

(2) $3y - 4 = 2x + 3$ and $y - 5 = x + 6$.

Solution.

$3y = 2x + 3 + 4$

$3y = 2x + 7$

$y = \frac{2}{3}x + \frac{7}{3}$

$M_1 = \frac{2}{3}$

$y - 5 = x + 6$

$y = x + 6 + 5$

$y = x + 11$ ($y = (m)x + c$

$M_2 = 1$

For the two lines to be perpendicular to each other, $M_1 M_2 = -1$

$\frac{2}{3} \cdot 1 = \frac{2}{3}$ ($\neq -1$)

Since $M_1 M_2$ is not equal -1, The pair of lines are **not** perpendicular to each other.

3. $x^2 + y^2 + 3xy - 11 = 0$ at the point $x=1, y=2$

Solution.

$$2x + 2y\frac{dy}{dx} + 3\left(x\frac{dy}{dx} + y\cdot 1\right) - 0 = 0$$

$$2x + 2y\frac{dy}{dx} + 3x\frac{dy}{dx} + 3y = 0$$

$$2x + \frac{dy}{dx}(2y + 3x) + 3y = 0$$

$$\frac{dy}{dx}\frac{(2y+3x)}{2y+3x} = \frac{-2x-3y}{2y+3x}$$

$$\frac{dy}{dx} = \frac{-2x-3y}{2y+3x}$$

$$m_1 = \frac{dy}{dx} = \frac{-2(1)-3(2)}{2(2)-3(1)} = \frac{-8}{7}$$

$$M_1 = \frac{-8}{7}$$

For Equation of tangent, $M_1 = M_2$

$$y - y_1 = M_1(x - x_1)$$

$$y - 2 = \frac{-8}{7}(x-1)$$

$$7y - 14 = -8x + 8$$

$$7y + 8x - 14 - 8 = 0$$

$7y + 8x - 22 = 0$ → Equation of the tangent.

For Equation of normal $M_2 = \frac{-1}{M_1}$

$$y - y_1 = \frac{-1}{M_1}(x - x_1)$$

$$y-2 = \frac{-1}{\frac{-8}{7}}(x-1)$$

$$y-2 = \frac{7}{8}(x-1)$$

$$8y-16 = 7x-7$$

$$8y-7x-16+7=0$$

$$8y-7x-9=0$$

∴ The Equation of the normal is $8y-7x-9=0$