NAME: ADELEKE MATHEUS ADEMOLA
Matric NO: 19/MHS01/029
Serial NO: 182
Department: Medicine and Surgery Date: 07-04-2020

Q Examine whether or not these pair of lines are perpendicular to each other
(A) $y-3x-2=0$ and $3y+x+9=0$ (B) $3y-4=2x+3$ and $y-5=x+6$

Solution

For two lines to be perpendicular to each other, then $m_1 m_2 = -1$

(A) $y-3x-2=0$ equation (i)

$3y+x+9=0$ equation ii

from equation (i);

$y = 3x+2$

Compare with $y=mx+c$

$\Rightarrow \quad m_1 = 3$

for equation (ii);

$y = \frac{-x}{3} + 3$. Compare with $y=mx+c$

$\Rightarrow \quad m = -1/3$

$m_1 \times m_2 = 3 \times -\frac{1}{3} = -1$

Since $m_1 m_2 = -1$, therefore, the pair of lines $y-3x-2=0$ and $3y+x+9=0$ are perpendicular to each other.

B $3y-4=2x+3$ and $y-5=x+6$

$\Rightarrow \quad y = \frac{2}{3}x + \frac{7}{3}$ $\Rightarrow \quad y = x+11$

Compare with $y=mx+c$ Compare with $y=mx+c$

$\Rightarrow \quad m_1 = \frac{2}{3}$ $\Rightarrow m_2 = 1$

$m_1 m_2 = \frac{2}{3} \times 1 = \frac{2}{3}$

Since $m_1 m_2$ is not equal to ($\neq$) -1, therefore the pair lines are not perpendicular to each other.

Find the equations of tangent and normal to the curve $x^2+y^2+3xy-11=0$ at the point $x=1, y=2$.

Solution

$$x^2+y^2+3xy-11=0$$

$$\frac{dy}{dx}=m \Rightarrow 2x+2y\frac{dy}{dx}+3\left(x\cdot\frac{dy}{dx}+y\cdot 1\right)-0=0$$

$$\frac{dy}{dx} \Rightarrow 2x+2y\frac{dy}{dx}+3x\frac{dy}{dx}+3y=0$$

$$\frac{dy}{dx}(2y+3x)=-2x-3y$$

$$\therefore \frac{dy}{dx}=\frac{-2x-3y}{2y+3x}$$

$$m=\left.\frac{dy}{dx}\right|_{(1,2)}=\frac{-2(1)-3(2)}{2(2)+3(1)}=\frac{-8}{7}=-1\frac{1}{7}$$

$$m=\frac{-8}{7}$$

$$x_1=1 \quad \text{and} \quad y_1=2$$

For equation of tangent, $y-y_1=m(x-x_1)$

$$\Rightarrow y-2=-8/7(x-1)$$

$$7y-14=-8x+8$$

$$7y+8x-14-8=0$$

$$\therefore 7y+8x-22=0 \quad \text{(Equation of tangent)}$$

For equation of Normal, $y-y_1=\frac{-1}{m}(x-x_1)$

$$\Rightarrow y-2=\frac{-1}{-8/7}(x-1)$$

$$y-2=\frac{7}{8}(x-1)$$

$$8y-16=7x-7$$

$$8y-7x-16+7=0$$

$$\therefore 8y-7x-9=0 \quad \text{(Equation of Normal)}$$

The equations of tangent and normal to the curve $x^2+y^2+3xy-11=0$ @ point $(1,2)$ are $7y+8x-22=0$ and $8y-7x-9=0$ respectively