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MECHANICAL ENGINEERING

19/ENG06/015

MAT 102

1) If $A = 2i - j$, $B = 3i + j - 11k$ and $C = 4i + 4j - 5k$, find

i) $-3A + 7B - 8C$

$$-3A = -3(2i - j) = -6i + 3j$$

$$7B = 7(3i + j - 11k) = 21i + 7j - 77k$$

$$-8C = -8(4i + 4j - 5k) = -32i - 32j + 40k$$

$$-3A + 7B - 8C = -6i + 3j + 21i + 7j - 77k - 32i - 32j + 40k$$

$$= -38i + 21i + 10j - 32j - 37k$$

$$= -17i - 22j - 37k$$

ii) $K = 2A + 4B - C$

$$K = 2(2i - j) + 4(3i + j - 11k) - (4i + 4j - 5k)$$

$$K = 4i + 12i - 4i - 2j + 4j - 4j - 44k + 5k$$

$$K = 12i - 2j - 39k$$

$$\text{Direction cosine of } k = \frac{12}{\sqrt{12^2 + (-2)^2 + (-39)^2}}$$

$$|K| = \frac{12}{\sqrt{144 + 4 + 1521}}$$

$$= \frac{12}{\sqrt{1669}}$$

$$= 0.294$$

$$\cos \alpha = \frac{12}{40.85} = l = 0.294$$

$$\cos \beta = \frac{-2}{40.85} = m = -0.049$$

$$\cos \gamma = \frac{-39}{40.85} = n = -0.955$$

iii) $A \times (B \times C)$

$$B \times C = \begin{pmatrix} i & j & k \\ 3 & 1 & -11 \\ 4 & 4 & -5 \end{pmatrix}$$

$$i \begin{pmatrix} 1 & -11 \\ 4 & -5 \end{pmatrix} - j \begin{pmatrix} 3 & -11 \\ 4 & -5 \end{pmatrix} + k \begin{pmatrix} 3 & 1 \\ 4 & 4 \end{pmatrix}$$

$$i[(1 \times (-5)) - (11 \times 4)] - j[(3 \times (-5)) - ((-11) \times 4)] + k[(3 \times 4) - (1 \times 4)]$$

$$i(-5 + 44) - j(-15 + 16) + k(12 - 4)$$

$$= 39i - j + 8k$$

$$A \times (B \times C) = \begin{pmatrix} i & j & k \\ 2 & -11 & 0 \\ 39 & -1 & 8 \end{pmatrix}$$

$$= i \begin{pmatrix} -11 & 0 \\ -1 & 8 \end{pmatrix} - j \begin{pmatrix} 2 & 0 \\ 39 & 8 \end{pmatrix} + k \begin{pmatrix} 2 & -11 \\ 39 & -1 \end{pmatrix}$$

$$i [((-11) \times 8) - (0 \times (-1))] - j [(2 \times 8) - (0 \times 39)] + k [(2 \times (-1)) - ((-11) \times 39)]$$

$$i(-88 - 0) - j(16 - 0) + k(-2 + 429)$$

$$= -88i - 16j + 427k$$

$$\text{iv) } (3A \times B) \cdot (A \times 2B)$$

$$3A \times B = \begin{pmatrix} i & j & k \\ 6 & -3 & 0 \\ 3 & 1 & -11 \end{pmatrix}$$

$$= i \begin{pmatrix} -3 & 0 \\ 1 & -11 \end{pmatrix} - j \begin{pmatrix} 6 & 0 \\ 3 & -11 \end{pmatrix} + k \begin{pmatrix} 6 & -3 \\ 3 & 1 \end{pmatrix}$$

$$= i [((-3) \times (-11)) - (0 \times 1)] - j [(6 \times (-11)) - (0 \times 3)] + k [(6 \times 1) - ((-3) \times 3)]$$

$$= 33i + 66j + 15k$$

$$A \times 2B = \begin{vmatrix} i & j & k \\ 2 & -1 & 0 \\ 6 & 2 & -22 \end{vmatrix}$$

$$i \begin{vmatrix} -1 & 0 \\ 2 & -22 \end{vmatrix} - j \begin{vmatrix} 2 & 0 \\ 6 & -22 \end{vmatrix} + k \begin{vmatrix} 2 & -1 \\ 6 & 2 \end{vmatrix}$$

$$i [((-1) \times (-22)) - (0 \times 2)] - j [(2 \times (-22)) - (0 \times 6)] + k [(2 \times 2) - ((-1) \times 6)]$$

$$= 22i + 44j + 10k$$

$$(3A \times B) \cdot (A \times 2B) = (33i + 66j + 15k) \cdot (22i + 44j + 10k)$$

$$= 726 + 2904 + 150$$

$$= 3780$$

$$v) A - 2B - C$$

$$= (2i - j) - (6i + 2j - 22k) - (4i + 4j - 5k)$$

$$A - 2B - C = -8i - 7j + 49k$$

2a) Perpendicular vectors: This is when the dot multiplication of two vectors A and b is equal to zero, i.e. $A \cdot B = 0$

2b) Coplanar vectors: This is when the scalar triple of three vectors is equal to zero, i.e. $A \cdot (B \times C) = 0$