

1) $\int \frac{dx}{x^2+7}$

let $u = x/\sqrt{7} \rightarrow du/dx = 1/\sqrt{7} \rightarrow dx = \sqrt{7} du$

$$\int \frac{1}{x^2+7} = \int \frac{\sqrt{7}}{7u^2+7} du$$

$$= \frac{1}{7} \int \frac{1}{u^2+1} du$$

$$\int \frac{1}{u^2+1} du$$

$$= \arctan(u)$$

$$\frac{1}{\sqrt{7}} \int \frac{1}{u^2+1} du = \frac{\arctan(u)}{\sqrt{7}}$$

recall $u = x/\sqrt{7}$

$$\arctan(u) = \arctan\left(\frac{x}{\sqrt{7}}\right)$$

Thus $\int \frac{dx}{x^2+7} = \frac{\arctan\left(\frac{x}{\sqrt{7}}\right)}{\sqrt{7}} + C$

2) $\int \frac{dx}{x^2+64}$

let $u = x/8 \rightarrow du/dx = 1/8 \rightarrow dx = 8 du$

$$\int \frac{dx}{x^2+64} = \int \frac{8}{64u^2+64} du$$

$$= \frac{1}{8} \int \frac{1}{u^2+1} du$$

Also $\int \frac{1}{u^2+1} du = \arctan(u)$

recall $u = x/8$

$$\arctan(u) = \arctan\left(\frac{x}{8}\right)$$

$$= \frac{\arctan\left(\frac{x}{8}\right)}{8} + C$$