

$$m_1 m_2 = -1$$

$$-\frac{5}{7} m_2 = -1$$

$$\therefore m_2 = \frac{7}{5}$$

$$y - y_1 = m_2 (x - x_1)$$

$$y - 2 = \frac{7}{5} (x - 1)$$

$$y - 2 = \frac{7x}{5} - \frac{7}{5}$$

$$y - 2 = \frac{7x - 7}{5}$$

$$5y - 10 = 7x - 7$$

$$5y - 7x - 9 = 0 \quad \text{eqn of normal}$$

$$x^2 + y^2 + 3xy - 11 = 0 \quad \text{at point } (1, 2)$$

Substit

$$\text{Differentiate } x^2 + y^2 + 3xy - 11 = 0$$

$$2x + 2y \frac{dy}{dx} + 3(x \frac{dy}{dx} + y) = 0$$

$$2x + 2y \frac{dy}{dx} + 3x \frac{dy}{dx} + 3y = 0$$

$$\frac{dy}{dx} (2y + 3x) = -2x - 3y$$

$$\frac{dy}{dx} = \frac{-2x - 3y}{2y + 3x}$$

$$\therefore \frac{dy}{dx} \Big|_{x=1, y=2} = 2$$

$$\frac{-2(1) - 3(2)}{2(2) + 3(1)}$$

$$= \frac{-2 - 6}{4 + 3} = \frac{-8}{7}$$

$$\therefore \frac{dy}{dx} = -\frac{8}{7}$$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{8}{7}(x - 1)$$

$$y - 2 = \frac{-8x + 8}{7}$$

$$y - 2 = \frac{-8x + 8}{7}$$

$$7(y - 2) = -8x + 8$$

$$7y - 14 = -8x + 8$$

$$7y - 14 + 8x + 8 = 0$$

$$7y + 8x - 22 = 0 \quad \text{eqn of tangent}$$

Name: OKWUMMO Dimebi Kizito
 Mats: 19/MHSO/328
 Course: MATHS 104

MBBS

* Examine whether or not these pairs of lines are perpendicular to each other

① $y - 3x - 2 = 0$ and $3y + x + 9 = 0$

② $3y - 1 = 2x + 3$ and $y - 5 = x + 6$

③ Find the equations of the tangent and normal to the circle $x^2 + y^2 + 32y - 11 = 0$ at $(1, 2)$

Ans

① $y - 3x - 2 = 0$

Rearrange into the form, $y = mx + c$

$y = 3x + 2$

$\therefore m_1 = 3$

Also, $3y + x + 9 = 0$

$\frac{3y}{3} = \frac{-x-9}{3}$

$y = \left(\frac{-1}{3}\right)x - 3 \quad m_2 = -\frac{1}{3}$

$\therefore m_1 m_2 = ?$

$3 \times -\frac{1}{3} = -1$

\therefore The lines are perpendicular

$(m_1 m_2 = -1)$

$$y = x + 1$$

$$y = mx + c$$

$$y = 1$$

$$m_1 m_2 = \frac{2}{3} \times 1 = \frac{2}{3}$$

$$\therefore m_1 m_2 \neq -1$$

Hence the lines are not perpendicular

③ $x^2 + y^2 + 3xy - 11 = 0$ at point (1, 2)

differentiate $x^2 + y^2 + 3xy - 11 = 0$

$$2x + 2y \frac{dy}{dx} + 3(x \frac{dy}{dx} + y) = 0$$

$$2x + 2y \frac{dy}{dx} + 3x \frac{dy}{dx} + 3y = 0$$

$$\frac{dy}{dx} (2y + 3x) = -2x - 3y$$

$$\frac{dy}{dx} = \frac{-2x - 3y}{2y + 3x}$$

$$\therefore \frac{dy}{dx} \Big|_{x=1, y=2}$$

$$= \frac{-2(1) - 3(2)}{2(2) + 3(1)}$$

$$= \frac{-2 - 6}{4 + 3} = \frac{-8}{7}$$

$$\therefore \frac{dy}{dx} = -\frac{8}{7}$$

$$y - y_1 = m_1 (x - x_1)$$

$$y - 2 = -\frac{8}{7} (x - 1)$$

$$y - 2 = \frac{8}{7} - \frac{8}{7}x$$

⑤ find the equations of the tangent and normal to the circle $x^2 + y^2 + 2x - 11 = 0$ at $(1, 2)$

Ans

$$\textcircled{1} \quad y - 3x - 2 = 0$$

rearrange into the form, $y = mx + c$

$$y = \textcircled{3}x + 2$$

$$\therefore m_1 = 3,$$

$$\text{Also, } 3y + 2x + 9 = 0$$

$$\frac{dy}{dx} = \frac{-x-9}{3}$$

$$y = \textcircled{\frac{-1}{3}}x - 3 \quad m_2 = -\frac{1}{3}$$

$$\therefore m_1 m_2 = ?$$

$3 \times -\frac{1}{3} = -1 \therefore$ The lines are perpendicular
($m_1 m_2 = -1$)

$$\textcircled{2} \quad 3y - 4 = 2x + 3$$

$$\frac{dy}{dx} = \frac{2x+7}{3}$$

$$y = \textcircled{\frac{2}{3}}x + \frac{7}{3} \quad m_1 = \frac{2}{3}$$

$$y = mx + c$$

$$m = \frac{2}{3}$$

$$y - 5 = x + 6$$

$$y = x + 11$$