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Examine whether or not these pair of lines are perpendicular to each other. (b)

(1) $y - 3x - 2 = 0$ and $3y + x + 9 = 0$

(2) $3y - 4 = 2x + 3$ and $y - 5 = x + 6$.

(3) Find the equations of the tangent and normal to the curve $x^2 + y^2 + 3xy - 11 = 0$ at the point $x = 1$, $y = 2$.

SOLUTION

(1) Make y the subject formula:

(a) $y = 3x + 2$; $m_1 = 3$

(b) $3y + x + 9 = 0$

$3y = -x - 9$

$y = \frac{-x}{3} - 3$; $\therefore m_2 = -\frac{1}{3}$

$m_1 \times m_2 = 3 \times -\frac{1}{3} = -1$ (Perpendicular)

∴ The lines $y - 3x - 2 = 0$ and $3y + x + 9 = 0$ are perpendicular

(2) Make y the subject formula:

(a) $3y - 4 = 2x + 3$

$3y = 2x + 3 + 4$

$3y = 2x + 7$

$y = \frac{2}{3}x + \frac{7}{3}$; $m_1 = 2/3$

(b) $y - 5 = x + 6$

$y = x + 6 + 5$

$y = x + 11$; $m_2 = 1$

$m_1 \times m_2 = 2/3 \times 1 = 2/3$ (Not perpendicular)

∴ The lines; $3y - 4 = 2x + 3$ and $y - 5 = x + 6$ are not perpendicular.

(c) $x^2 + y^2 + 3xy - 11 = 0$

$2x + 2y\frac{dy}{dx} + 3\left[x \cdot \frac{dy}{dx} \cdot 1 + 1 \cdot y\right] = 0$

$2x + 2y\frac{dy}{dx} + 3x\frac{dy}{dx} + 3y = 0$

$2y\frac{dy}{dx} + 3x\frac{dy}{dx} = -2x - 3y$

$\frac{dy}{dx}(2y + 3x) = -2x - 3y$

$\frac{dy}{dx} = \frac{-2x - 3y}{2y + 3x}$

$\frac{dy}{dx}$ at the point x=1 and y=2=

$$\frac{dy}{dx} = \frac{-2(1) - 3(2)}{2(2) + 3(1)} = \frac{-2-6}{7} = \frac{-8}{7}$$

$\therefore m = -8/7$.

a) Equation of tangent: $y - y_1 = m(x - x_1)$

$y - 2 = -8/7(x - 1)$

$7y - 14 = -8(x - 1)$

$7y - 14 = -8x + 8$

$7y + 8x - 14 - 8 = 0$

Equation of tangent $= 7y + 8x - 22 = 0$

b) Equation of normal; $y - y_1 = -1/m(x - x_1)$

$y - 2 = -1/-8/7(x - 1)$

$y - 2 = 7/8(x - 1)$

$8y - 16 = 7(x - 1)$

$8y - 16 = 7x - 7$

$8y - 7x - 16 + 7 = 0$

Equation of normal $= 8y - 7x - 9 = 0$.