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MAT 104

Mechanical Engineering S/N: 120

1) $y = \dfrac{(x+1)^2 (x-2)^{1/2}}{(2x-1)(x-3)^{4/3}}$

$\ln y = \ln(x+1)^2 + \ln(\sqrt{x-2}) - \ln(2x-1) - \ln(x-3)^{4/3}$

$\dfrac{1}{y}\cdot\dfrac{dy}{dx} = \dfrac{1}{(x+1)^2}\cdot 2(x+1) + \dfrac{1}{\sqrt{x-2}}\cdot\dfrac{1}{2}(x-2)^{-1/2} - \dfrac{1}{2x-1}\cdot 2 - \dfrac{1}{(x-3)^{4/3}}\cdot \frac{4}{3}(x-3)^{1/3}$

$\dfrac{1}{y}\cdot\dfrac{dy}{dx} = \dfrac{2}{(x+1)} + \dfrac{1}{2(\sqrt{x-2})(\sqrt{x-2})} - \dfrac{2}{2x-1} - \dfrac{4}{3}(x+3)^{1/2 - 4/3}$

$\dfrac{dy}{dx} = y\left[\dfrac{2}{x+1} + \dfrac{1}{2(\sqrt{x-2})(\sqrt{x-2})} - \dfrac{2}{2x-1} - \dfrac{3}{(3x-9)^{5/6}}\right]$

2) $y = \dfrac{3e^x \sin 2x}{k^{5/2}}$

$\ln y = \ln(3e^x) + \ln(\sin 2x) - \ln(k^{5/2})$

$\dfrac{1}{y}\cdot\dfrac{dy}{dx} = \dfrac{1}{3e^x}\cdot 3e^x + \dfrac{1}{\sin 2x}\cdot 2\cos 2x - \dfrac{1}{k^{5/2}}\cdot \frac{5}{2}k^{3/2}$

$\dfrac{1}{y}\,\dfrac{dy}{dx} = 1 + \dfrac{2\cos 2x}{\sin} \cdot \dfrac{5}{2}k^{3/2 - 5/2}$

$\dfrac{dy}{dx} = y\left[1 + \dfrac{2\cos 2x}{\sin 2x} - \dfrac{5}{2}k^{-1}\right]$

$\dfrac{dy}{dx} = \dfrac{3e^x \sin 2x}{k^{5/2}}\left[1 + \dfrac{2\cos 2x}{\sin 2x} - \dfrac{5}{2k}\right]$

3) $4\sec^2(3m+1)$

$4\int \sec^2(3m+1)\,dm$

let $u = 3m+1$

$\frac{du}{dm} = 3$

$\frac{du}{3} = \frac{3\,dm}{3}$

$dm = \frac{du}{3}$

$4\int \sec^2(u)\,du/3$

$\frac{4}{3}\tan u + c$

$= \frac{4}{3}\tan(3m+1) + c$

4) $\int 2t\,(3t^2-1)^{1/2}\,dt$

Let $u = \sqrt{3t^2-1}$

$u^2 = 3t^2-1$

$3t^2 = u^2+1 \qquad t^2 = \frac{u+1}{3}$

$t = \sqrt{\frac{u^2+1}{3}}$

$\frac{dt}{du} = \frac{1}{2}\left(\frac{u^2+1}{3}\right)^{-1/2} \cdot \frac{2u}{3}$

$dt = \frac{u\,du}{3}\left(\frac{u^2+1}{3}\right)^{-1/2}$

$$\frac{2}{3}\int v^2\,dv = \frac{2v^3}{9} + C$$

$$= \frac{2(3t^2-1)^{3/2}}{9} + C$$

5) $\int \frac{2x}{\sqrt{4x^2-1}}\,dx$

$v = \sqrt{4x^2-1}$

$v^2 = 4x^2-1$

$4x^2 = v^2+1$

$x^2 = \frac{v^2+1}{4}$

$x = \sqrt{\frac{v^2+1}{4}}$

$$\frac{dx}{dv} = \frac{1}{2}\left(\frac{v^2+1}{4}\right)^{-1/2} \cdot \frac{v}{2}$$

$$\frac{dx}{dv} = \frac{v}{4}\left(\frac{v^2+1}{4}\right)^{-1/2}$$

$$\int 2\left(\frac{v^2+1}{4}\right)^{1/2} \cdot \frac{v\,dv}{4}\left(\frac{v^2+1}{4}\right)^{-1/2}$$

$$\frac{1}{2}\int\left(\frac{v^2+1}{4}\right)^{1/2-1/2} dv$$

$$= \frac{1}{2}\int dv = \frac{v}{2} + C$$

$$= \frac{\sqrt{4x^2-1}}{2} + C$$