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MATRIC NO: 19/SCI01/015

ASSIGNMENT

Find the integral of the following

(a) $\int \frac{dx}{x^2+7}$

Solution

$$\int \frac{dx}{x^2+7} = \int \frac{dx}{7\left(\frac{x^2}{7}+1\right)} = \frac{1}{7}\int \frac{dx}{\frac{x^2}{7}+1} = \frac{1}{7}\int \frac{dx}{\left(\frac{x}{\sqrt{7}}\right)^2+1}$$

$u = \frac{x}{\sqrt{7}}$

(Rationalize)

$u = \frac{x}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}x}{7}$

$u = \frac{\sqrt{7}x}{7}$

$\frac{du}{dx} = \frac{\sqrt{7}}{7}$

$dx = \frac{7}{\sqrt{7}} du$

$$= \frac{1}{7}\int \frac{1}{u^2+1} dx = \frac{1}{7}\int \frac{1}{u^2+1} \cdot \frac{7}{\sqrt{7}} du = \frac{7}{7\sqrt{7}}\int \frac{1}{u^2+1} du$$

Recall, $\int \frac{1}{u^2+1} du$ is a standard integral, so it equates to arctan

$= \frac{7}{7\sqrt{7}} \arctan(u) + c$. Recall $u = \sqrt{7}x/7$

$= \frac{\sqrt{7}}{7} \arctan(\sqrt{7}x/7) + c$

$= \frac{\sqrt{7} \times \arctan(\sqrt{7}x/7)}{7} + c$

(b) $\int \frac{dx}{x^2+64}$

Solution

$$\int \frac{dx}{x^2+64} = \int \frac{dx}{64\left(\frac{x^2}{64}+1\right)} = \frac{1}{64}\int \frac{dx}{\frac{x^2}{64}+1} = \frac{1}{64}\int \frac{dx}{\left(\frac{x}{8}\right)^2+1}$$

$u = \frac{x}{8}$

$\frac{du}{dx} = \frac{1}{8}$

$dx = 8\,du$

$$= \frac{1}{64}\int \frac{1}{u^2+1}\,dx = \frac{1}{64}\int \frac{1}{u^2+1} \cdot 8\,du = \frac{\cancel{8}^{1}}{\cancel{64}_{8}}\int \frac{1}{u^2+1}\,du$$

Recall, $\int \frac{1}{u^2+1}\,du$ is a standard integral, so it equates to:

$\arctan(u)$

$$= \frac{1}{8}\int \frac{1}{u^2+1}\,du = \frac{1}{8}\arctan(u), \text{ Recall } u = \frac{x}{8}$$

$$= \frac{\arctan(x/8)}{8} + C$$