

$$\textcircled{1} \quad y = \frac{1}{x-2}$$

- The function is defined for all

real numbers except $x = 2$

- The domain is ^{The} set of real numbers except $x = 2$

- The codomain is the set of real numbers except $y = 0$

$$\textcircled{2} \quad k = 100$$

$$\frac{ds}{dv} = \frac{1}{v}$$

$$\textcircled{3} \quad 2x - 3y - 2 = 0$$

$$-3y = 2 - 2x$$

$$y = \frac{2 - 2x}{-3}$$

$$y = \frac{2x+2}{3}, \frac{2(x+1)}{3}$$

$$y = \frac{2x+2}{3}, \frac{2(x+1)}{3}$$

$$\textcircled{4} \quad x^2 + y^2 = 4$$

$$y^2 = 4 - x^2$$

$$y = \pm \sqrt{4 - x^2}$$

$$\textcircled{4} \quad \text{find } \frac{dp}{dt}, p = \sin^{-1} t + \frac{1}{\sin t},$$

$$t = \sin p - \rightarrow$$

$$\frac{dt}{dp} = \cos p, \frac{dp}{dt} = \frac{1}{\cos p}$$

$$\text{Recall } \cos^2 y + \sin^2 y = 1$$

$$\cos y = \pm \sqrt{1 + \sin^2 y}$$

$$t = \sin p \therefore \cos p = \sqrt{1 + t^2}$$

$$\text{Hence } \frac{dp}{dt} = \frac{1}{1+t^2}$$

$$\textcircled{8} \quad f(x) = 2x^3 - 5, g(x) = 4x - 2$$

$$f \circ g(x) = 2(4x - 2)^2 - 5$$

$$= 2(16x^2 - 16x + 4) - 5$$

$$= 32x^2 - 32x + 8 - 5$$

$$= 32x^2 - 32x + 3$$

$$g \circ f(x) = 4(2x^2 - 5) - 2$$

$$= 8x^2 - 20 - 2$$

$$= 8x^2 - 22.$$

\textcircled{6}

Show that $f(x) = f_1(x) + f_2(x)$

$$f(x) = 3x^2 - 2x + 1$$

$$f_1(x) = \underline{\underline{f_1(x) + f_2(x)}} \\ f_2(x) = 3x^2 - 2x + 1 + (3x^2 + 2x + 1)$$

$$= \frac{6x^2 + 2}{2} = 3x^2 + 1$$

$$f_1(x) = \frac{3x^2 - 2x + 1 + (3x^2 + 2x + 1)}{2}$$

$$= \frac{6x^2 + 2}{2} = 3x^2 + 1$$

$$f_2(x) = \frac{3x^2 - 2x + 1 - (3x^2 + 2x + 1)}{2} \\ = -4x \frac{2}{2} = -2x$$

$$f_1(x) + f_2(x) = 3x^2 + 1 - 2x \\ = 3x^2 - 2x + 1$$

\textcircled{7} Differentiate $y = \cos x$.

$$y = \frac{dy}{dx} = \cos(x + \Delta x)$$

$$\frac{dy}{dx} = \cos(\omega x + \omega \Delta x) - \cos \omega x \quad (y = \cos \omega x)$$

Recall

$$\cos(A+B) - \cos(A-B) = -2 \sin A \sin B$$

$$A+B = \omega x + \omega \Delta x \quad \text{--- (3)}$$

$$A-B = \omega x \quad \text{--- (4)}$$

$$2A = 2\omega x + \omega \Delta x \quad \text{--- (5)} \\ \text{and } B = \omega x / 2$$

$$A = \frac{2\omega x + \omega \Delta x}{2}$$

$$I) f(x) = x^2 + 5x + 6 \text{ evaluate at } x=0$$

7) Continuation

$$\begin{aligned} A &= x + \frac{dx}{2} \\ &\approx \cos(x) - \cos(x) \\ &= 2\sin(x + \frac{\delta x}{2}) \end{aligned}$$

$$= 2\sin(n + \frac{\delta x}{2})\sin(\delta x/2)$$

Dividing through by δx

$$\frac{dy}{dx} = -2\sin(x + \frac{\delta x}{2})\sin(\frac{\delta x}{2})$$

$$\frac{dy}{dx} = -\sin(x + \frac{\delta x}{2})\sin(\frac{\delta x}{2})$$

$$= -\sin(x + \frac{\delta x}{2}) \times \sin(\frac{\delta x}{2})$$

Taking limit $\delta x \rightarrow 0$

$$\delta x \rightarrow 0 \quad \frac{\sin(\delta x/2)}{\delta x/2} = 1$$

$$\frac{dy}{dx} = -\sin(x + 0) \times 1$$

$$\sin \rightarrow 0$$

$$\frac{dy}{dx} = -\sin x$$

$$8) y = 3t^2, x = t^2$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= \frac{dy}{dt} \div \frac{dx}{dt}$$

$$\frac{dy}{dt} = 6t, \frac{dx}{dt} = \frac{-2}{t^3}$$

$$\frac{dy}{dx} = 6t \div -2$$

$$= 6t \times \frac{-2}{t^3} = \frac{6x - 2}{t^2} = \frac{12}{t^2}$$

$$\frac{dI}{dx} = \frac{-12}{t^2}$$

$$9) y = x^2 \cos 2x \cdot e^{4x}$$

Sol

Taking loge on both sides

$$\ln y = \ln x^2 + \ln \cos 2x + \ln e^{4x}$$

Differentiating both sides

$$\frac{1}{y} \frac{dy}{dx} = \frac{1}{x^2}(2x) + \frac{1}{\cos 2x}(-2\sin 2x)$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{2}{x} - \frac{2\sin 2x}{\cos 2x} + 4$$

Multiplying both by y

$$\frac{dy}{dx} = y \left(\frac{2}{x} - \frac{2\sin 2x}{\cos 2x} + 4 \right)$$

$$= x^2 \cos 2x e^{4x} \frac{2}{x} - \frac{2\sin 2x}{\cos 2x} + 4$$

$$10) y = \sin(3x^3 + 5)$$

$$\text{let } u = 3x^3 + 5$$

$$\frac{dy}{dx} = \cos x$$

$$\frac{du}{dx} = 9x^2$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= \cos x \times 9x^2$$

$$= 9x^2 \cos x$$

$$= 9x^2 \cos 3x^3 + 5$$