MATHS

1. For what value of x is function $y = 1/(x-2)$ defined? State the domain and Co-domain

Ans – $y = 1/x - 2$. Function is defined for all real numbers except $x=2$

domain: all real numbers of x except 2

Co domain: all real numbers of y

2. If $k = \ln v$ differentiate k

$$\frac{d}{dk}(\ln v) = \frac{1}{v}$$

3. $\frac{dy}{dx}$ if $y = 3t^2$ $x = 1/t^2$

$$\frac{dy}{dx} = \frac{dy}{dt} - \frac{dt}{dx}$$

$$= \frac{dy/dt}{dx/dt}$$

$$\frac{dy}{dt} = 6t$$

$$\frac{\cancel{dy}}{dt} = -2t^{-3}$$

$$\frac{dy}{dx} = \frac{6t}{1/t^2}$$

$$= -3t^4$$

10) $y = \sin(3x^3 + 5)$

$u = 3x^3 + 5$

$y = \sin u$

$\frac{dy}{dx} = 9x2$

$\frac{dy}{du} = \cos u$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$= \cos u \times 9x2$

$\frac{dy}{dx} = 9x^2 \cos u$

but $u = 3x3 + 5$

$\frac{dy}{dx} = 9x^2 \cos(3x3 + 5)$

7). $y = d \quad x$

$y + \delta y = \cos(x + \delta x)$

Subtract y from both sides

$\delta y = \cos(x + \delta x) - y$

but $y = \cos x$

$\delta y = \cos(x + \delta x) - \cos x$ — (i)

~~[illegible]~~ Considering

$\cos(A + B) = \cos A \cos B - \sin A \sin B$

$\cos A - B = \cos A \cos B + \sin A \sin B$

$\cos(A+B) - \cos(A-B) = -2 \sin A \sin B$ — (ii)

compare i and ii

let $A + B = x + \delta x$ — 1

$A - B = x$ —— 2

Adding i and ii

$2A = 2x + \delta x$

$A = \dfrac{2x + \delta x}{2}$

$\left.\begin{array}{l} A = x + \delta x/2 \\ B = \delta x/2 \end{array}\right\}$ (iii)

Compare eqn (iii) and (ii)

$\cos(x + \delta x) - \cos x = -2\sin(x + \delta x/2)$

$\delta y = -2\sin(x + \delta x/2)\ \sin(\delta x/2)$

Dividing through by δx

$\dfrac{\delta y}{\delta x} = -2\sin(x + \delta x/2)\ \sin(\delta x/2)\ /\ \delta x$

$= -\sin\left(x + \dfrac{\delta x}{2}\right)\ \sin(\delta x/2)\ /\ \delta x/2$

Take lim

$\lim \dfrac{\delta y}{\delta x} = \dfrac{dy}{dx} = -\sin(x + 0) \cdot 1$

$\delta x \to 0 \quad = -\sin x$

$\lim\limits_{\delta x \to 0} \dfrac{\delta y}{\delta x} = dy/dx = -\sin x$

6) $F(x) = 3x^2 - 2x + 1 = 0$

$F_e x = F(x) + f(-x)/2$

$F(\cancel{-x}) = 3(-x)^2 - 2(-x) + 1$

$= 3x^2 + 2x + 1$

$F_e(x) = 3x^2 - 2x + 1 + 3x^2 + 2x + 1\ /2$

$= 6x^2 + 2/2$

$= 3x^2 + 1$

$F_o(x) = F(x) - f(-x)/2$

$$= \frac{3x^2 - 2x + 1 - [3x^2 + 2x + 1]}{2}$$

$= 3x^2 - 2x + 1 - 3x^2 - 2x - 1 \; / 2$

$$= \frac{-4x}{2}$$

$= -2x$

$F_x = F_0(x) + F_0(x)$

$F_x = 3x^2 + 1 + (-2x)$

$F_x = 3x^2 - 2x + 1$

5) $f(x) = 2x^2 - 5$

$g(x) = 4x - 2$

$fog(x)$

$fog(x) = 2(4x-2)^2 - 5$

$= 2\{(4x-2)(4x-2)\} - 5$

$= 2\{16x^2 - 16x + 4\} - 5$

$fog(x) = 32x^2 - 32x + 8 - 5$

$32x^2 - 32x + 3$

$gof = g(2x^2 - 5)$

$4(2x^2 - 5) - 2$

$= 8x^2 - 20 - 2$

$= 8x^2 - 22.$

9) ~~Find~~ $\ln y = \ln x^2 + \ln \cos 2x + \ln e^{4x}$

$d/dx(\ln y) = d/dx\,(\ln x^2) + d/dx$

$(\ln(\cos 2x)) + d/dx\,(\ln e^{4x})$

$1/y\,(dy/dx) = 1/x^2\,(2x) + 1(-\sin 2x)/\cos 2x + 1(e^{4x})/$

$$\frac{dy}{dx} = \frac{2x}{x^2} - \frac{\sin 2x + 1}{\cos 2x}$$

$$= \frac{dy}{dx} = x^2 \cos 2x\, e^{4x}\, \frac{(2x^2 - \sin 2x + 1)}{(x \cos 2x)}$$

3a) $2x - 3y - 2 = 0$

$2x - 3y = 2$

$\frac{dy}{dx}(x) - \frac{d2}{dx}(3y) = \frac{d}{dx}(0)$

$2 - 3\frac{dy}{dx} = 0$

$2 = 3\frac{dy}{dx}$

$\frac{dy}{dx} = 2/3$

3b) $x^2 + y^2 = 4$

$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = 4$

$2x + 2y \cdot \frac{dy}{dx} = 0$

$2y\frac{dy}{dx} = -2x$

$\frac{dy}{dx} = -\frac{2x}{2y}$