Name: Bamigboye Olumide Oluwaseun Matric No: 19/MHS11/038
College: Medical and health Sciences Course code: MAT104
Dept: Pharmacy

1. For what value of x is the function $y = \frac{1}{x-2}$ defined? State the domain and co domain

Solution

$y = \frac{1}{x-2}$

The function $y = \frac{1}{x-2}$ is defined for all real nos except $x = 2$

Domain: all real numbers except $x = 2$

Co-domain: all real numbers except $y = 0$

2 If $k = \ln V$, differentiate k

Solution

$k = \ln V$

$\frac{dk}{dV} = \frac{1}{V}$

$\frac{d}{dk}(\ln V) = \frac{1}{V}$ //

3 Express y as an explicit function of x in the following

a) $2x - 3y - 2 = 0$

$2x - 2 = 3y$

$y = \frac{2x-2}{3}$ //

b $x^2 + y^2 = 4$

$y^2 = 4 - x^2$

$y = \sqrt{4 - x^2}$ //

4 If $p = \sin^{-1} b$, find the derivative of p

Solution

$\frac{dp}{db}, p = \frac{b}{\sin}$

$b = \sin p$ ——— *

$\frac{dt}{dp} = \cos p$

$\frac{dp}{dt} = \frac{1}{\cos p}$

Recall, $\cos^2 y + \sin^2 y = 1$

$\cos y = \pm\sqrt{1+\sin^2 y}$

$t = \sin p$

$\therefore \cos p = \sqrt{1-t^2}$ //

6. If $f(x) = 3x^2 - 2x + 1 = 0$ show that $f_e(x) + f_o(x) = f(x)$

$f(x) = 3x^2 - 2x + 1$

$f_e(x) = \frac{f(x) + f(-x)}{2}$

$f(-x) = 3(-x)^2 - 2(-x) + 1$

$= 3x^2 + 2x + 1$

$f_e(x) = \frac{3x^2 - 2x + 1 + (3x^2 + 2x + 1)}{2}$

$= \frac{6x^2 + 2}{2} = 3x^2 + 1$

$f_o(x) = \frac{3x^2 - 2x + 1 - (3x^2 + 2x + 1)}{2}$

$= \frac{-4x}{2} = -2x$

$f_e(x) + f_o(x) = 3x^2 + 1 - 2x$

$= 3x^2 - 2x + 1$ //

5 If $f(x) = 2x^2 - 5$ and $g(x) = 4x - 2$, find $fog(x)$ and $gof(x)$

Solution

$f(x) = 2x^2 - 5$

$g(x) = 4x - 2$

$fog(x) = 2(4x-2)^2 - 5$

$= 2(16x^2 - 16x + 4) - 5$

$= 32x^2 - 32x + 8 - 5$

$= 32x^2 - 32x + 3$

$gof(x) = 4(2x^2 - 5) - 2$

$8x^2 - 20 - 2$

$= 8x^2 - 22$ //

7) Differentiate $y = \cos x$ from first principle

$y + \delta y = \cos(x + \delta x)$

$\delta y = \cos(x + \delta x) - \cos x$ —— i

Recall

$\cos(A+B) - \cos(A-B) = -2\sin A \sin B$ —— ii

Comparing eqn (i) & (i)

$A + B = x + \delta x$ —— iii

$A - B = x$ —— iv

Adding eqn (iii) & (iv) & Subtracting eqn (iii) & (iv)

$2A = 2x + \delta x$ & $B = \delta x/2$

$A = \frac{2x + \delta x}{2}$ $A = \frac{x + \delta x}{2}$

Comparing eqn (i) & (ii)

$\delta y = \cos(x + \delta x) - \cos x$

$= -2\sin\left(\frac{x + \delta x}{2}\right)\sin\left(\frac{\delta x}{2}\right)$

Dividing through by δx

$\frac{\delta y}{\delta x} = \frac{-2\sin\left(\frac{x + \delta x}{2}\right)\sin\left(\frac{\delta x}{2}\right)}{\delta x}$

$$\frac{\delta y}{\delta x} = \frac{-\sin\left(x+\frac{\delta x}{2}\right)\left(\frac{\delta x}{2}\right)}{\delta x}$$

$$= -\sin\left(x+\frac{\delta x}{2}\right) \times \frac{\sin\left(\frac{\delta x}{2}\right)}{\frac{\delta x}{2}}$$

taking limit $\delta x \to 0$

$$\lim_{\delta x \to 0} \frac{\sin \frac{\delta x}{2}}{\frac{\delta x}{2}} = 1$$

$$\frac{\delta y}{\delta x} = -\sin\left(x+0/2\right) \times 1$$

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = -\sin x //$$

8 Find $\frac{dy}{dx}$ if $y = 3t^2$, $3x = \frac{1}{t^2}$

Solution

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= \frac{dy}{dt} \div \frac{dx}{dt}$$

$$\frac{dy}{dt} = 6t \; ; \; \frac{dx}{dt} = \frac{-2}{t^3}$$

$$\frac{dy}{dx} = 6t \div -2/t^3$$

$$= 6t \times \frac{-2}{1/3}$$

$$= 6x\frac{-2}{t^2} = \frac{-12}{t^2}$$

$$\frac{dy}{dx} = \frac{-12}{t^2} //$$

9). Find dy/dx if $y = x^2 \cos 2x e^{4x}$

Solution

taking $\log_e$ from both sides

$$\ln y = \ln x^2 + \ln \cos 2x + \ln e^{4x}$$

Differentiating both x

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x^2}(2x) + \frac{1}{\cos 2x}(-2\sin 2x) + 4$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x} - \frac{2\sin 2x}{\cos 2x} + 4$$

Multiply both sides by y

$$\frac{dy}{dx} = y\left(\frac{2}{x} - \frac{2\sin 2x}{\cos 2x} + 4\right)$$

$$= x^2 \cos 2x e^{4x} \times \frac{2}{x} - \frac{2\sin 2x}{\cos 2x} + 4$$

10). Given that $y = \sin(3x^3+5)$, find the derivative of y

Solution

$y = \sin(3x^3+5)$

Let $t = 3x^3+5$

$$\frac{dy}{dt} = \cos t$$

$$\frac{dt}{dx} = 9x^2$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= \cos t \times 9x^2$$

$$= 9x^2 \cos t$$

$$= 9x^2 \cos 3x^3 + 5$$