

MATHS

1. For what value of x is function $y = \sqrt{x-2}$ defined? State the domain and Co-domain

Ans - $y = \sqrt{x-2}$. Function is defined for all real numbers except $x=2$
domain: all real numbers of x except 2
Co-domain: all real numbers of y

2. If $k = \ln x$ differentiate k

$$\frac{d}{dk} (\ln x) = \frac{1}{x}$$

8. $\frac{dy}{dx}$ if $y = 3t^2$ and $x = \frac{1}{t^2}$

$$\begin{aligned}\frac{dy}{dx} &= \frac{dy}{dt} \cdot \frac{dt}{dx} \\ &= \frac{dy/dt}{dx/dt}\end{aligned}$$

$$\frac{dy}{dt} = 6t$$

$$\frac{dx}{dt} = -2t^{-3}$$

$$\begin{aligned}\frac{dy}{dx} &= \frac{6t}{\frac{1}{t^2}} \\ &= -3t^4\end{aligned}$$

$$10) y = \sin(3x^3 + 5)$$

$$u = 3x^3 + 5$$

$$y = \sin u$$

$$\frac{dy}{dx} = 9x^2$$

$$\frac{dy}{du} = \cos u$$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \cos u \cdot 9x^2$$

$$\frac{dy}{dx} = 9x^2 \cos u$$

$$\text{but } u = 3x^3 + 5$$

$$\frac{dy}{dx} = 9x^2 \cos(3x^3 + 5)$$

$$7) y = \cos x$$

$$y + S_y = \cos(x + S_x)$$

Subtract y from both sides

$$S_y = \cos(x + S_x) - y$$

$$\text{but } y = \cos x$$

$$S_y = \cos(x + S_x) - \cos x \quad \text{--- (1)}$$

~~Consider~~ Consider from Trig

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\cos(A+B) - \cos(A-B) = -2\sin A \sin B \quad \text{--- (1)}$$

Compare 1 and 1

$$\text{let } A+B = x + S_x \quad \text{--- 1}$$

$$A-B = x \quad \text{--- 2}$$

Adding i and ii

$$2A = 2x + Sx$$

$$A = \frac{2x + Sx}{2}$$

$$\left. \begin{aligned} A &= x + Sx/2 \\ B &= Sx/2 \end{aligned} \right\} \text{(iii)}$$

Compare eqn (iii) and (ii)

$$\cos(x + Sx) - \cos x = -2\sin(x + Sx/2)$$

$$S_y = -2\sin(x + Sx/2) \sin(Sx/2)$$

Dividing through by S_y

$$\frac{S_y}{Sx} = -2\sin(x + Sx/2) \sin(Sx/2) / Sx$$

$$= -\sin(x + \frac{Sx}{2}) \sin(\frac{Sx}{2}) / \frac{Sx}{2}$$

Take lim

$$\lim_{Sx \rightarrow 0} \frac{S_y}{Sx} = \frac{dy}{dx} = -\sin(x+0) \cdot 1$$

$$Sx \rightarrow 0 = -\sin x$$

$$\lim_{Sx \rightarrow 0} \frac{S_y}{Sx} = \frac{dy}{dx} = -\sin x$$

$$b) F(x) = 8x^2 - 2x + 1 = 0$$

$$F_x = F(x) + f(x)/2$$

$$F(-x) = 8(-x)^2 - 2(-x) + 1 \\ = 8x^2 + 2x + 1$$

$$F_c(x) = 8x^2 - 2x + 1 + 8x^2 + 2x + 1 / 2$$

$$= 6x^2 + 2 / 2$$

$$= 3x^2 + 1$$

$$T_0(x) = F(x) - f(x)/2$$

$$= \frac{8x^2 - 2x + 1 - [3x^2 + 2x + 1]}{e}$$

$$= \frac{5x^2 - 2x + 1 - 3x^2 - 2x - 1}{2}$$

$$= \frac{-4x}{2}$$

$$= -2x$$

$$F_x = F_0(x) + F_0(x)$$

$$F_x = 8x^2 + 1 + (-2x)$$

$$F_x = 8x^2 - 2x + 1$$

$$5) f(x) = 2x^2 - 5$$

$$g(x) = 4x - 2$$

$$f \circ g(x)$$

$$f \circ g(x) = 2(4x - 2)^2 - 5$$

$$= 2[(4x - 2)(4x - 2)] - 5$$

$$= 2[16x^2 - 16x + 4] - 5$$

$$f \circ g(x) = 32x^2 - 32x + 8 - 5$$

$$32x^2 - 32x + 3$$

$$g \circ f = g(2x^2 - 5)$$

$$= 4(2x^2 - 5) - 2$$

$$= 8x^2 - 20 - 2$$

$$= 8x^2 - 22$$

$$9) \ln y = \ln x^2 + \ln \cos 2x + \ln e^{4x}$$

$$\frac{d}{dx} (\ln y) = \frac{d}{dx} (\ln x^2) + \frac{d}{dx}$$

$$(\ln \cos 2x) + \frac{d}{dx} (\ln e^{4x})$$

$$\frac{1}{y} \left(\frac{dy}{dx} \right) = \frac{1}{x^2} (2x) + \frac{1}{\cos 2x} (-\sin 2x) + \frac{1}{e^{4x}} (4e^{4x})$$

$$\frac{dy}{dx} = \frac{2x}{x^2} - \frac{\sin 2x}{\cos 2x} + 4$$

$$= \frac{dy}{dx} = \frac{2x}{x^2} \cos 2x e^{4x} - \frac{\sin 2x}{\cos 2x} + 4$$

$$3a) \quad 2x - 3y - 2 = 0$$

$$2x - 3y = 2$$

$$\frac{dy}{dx} (2) - \frac{d}{dx} (3y) = \frac{d}{dx} (0)$$

$$2 - 3 \frac{dy}{dx} = 0$$

$$2 = 3 \frac{dy}{dx}$$

$$\frac{dy}{dx} = 2/3$$

$$3b) \quad x^2 + y^2 = 4$$

$$\frac{d}{dx} (x^2) + \frac{d}{dx} (y^2) = 4$$

$$2x + 2y \cdot \frac{dy}{dx} = 0$$

$$2y \frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y}$$