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Examine whether or not these pair of lines are perpendicular to each other. 1) $y - 3x - 2 = 0$ and $3y + x + 9 = 0$. 2) $3y - 4 = 2x + 3$ and $y - 5 = x + 6$ 3) Find the equations of the tangent and normal to the curve x square + y square + $3xy - 11 = 0$ at the point $x = 1, y = 2$

Solution.

1) $y - 3x - 2 = 0$

$3y + x + 9 = 0$

For the lines to be perpendicular when $M_1 M_2 = -1$

$y - 3x - 2 = 0$

making y the subject of the formula.

$y = +3x + 2$

$y = 3x + 2$

By comparison with $y = mx + c \Rightarrow M_1 = 3$

$3y + x + 9 = 0$

making y the subject of the formula.

$3y = -x - 9$

$y = \frac{-x}{3} \quad \frac{-9}{3}$

$y = \frac{-1x}{3} - 3$

$y = mx + c \quad M_2 = -1/3$

$m_1 M_2 = -1$ for perpendicularity.

$3 \times -1/3 = -1$ Since $M_1 M_2 = -1$ then the lines $y - 3x - 2 = 0$ and $3y + x + 9 = 0$ Are Perpendicular

2) $3y - 4 = 2x + 3$ — ①

$y - 5 = x + 6$ — ②

making y the subject of the formula in 1

$3y = 2x + 3 + 4$

PTO

$3y = 2x + 7$

$y = \frac{2x}{3} + \frac{7}{3}$

By comparing with $y = mx + c$

$M_1 = 2/3$

making y the subject of formula in 2.

$y - 5 = x + 6$

$y = x + 6 + 5$

$y = x + 11$

By comparing with $y = mx + c$

$M_2 = 1$

But for the lines to be perpendicular;

$M_1 M_2 = -1$

$M_1 M_2 = 2/3 \times 1 = 2/3$

$M_1 M_2 \neq -1$

Hence the lines $3y - 4 = 2x + 3$ and $y - 5 = x + 6$ are not perpendicular.

3) $x^2 + y^2 + 3xy - 1 = 0$. $(x=1, y=2)$

$m = dy/dx$

$x^2 + y^2 + 3xy - 11 = 0$

dy/dx

$2x + 2y\, dy/dx + 3(x \cdot \frac{dy}{dx} + y \cdot 1) = 0$

$2x + 2y \frac{dy}{dx} + 3\left(x \cdot \frac{dy}{dx} + y\right) = 0$

$2x + 2y \frac{dy}{dx} + 3x \frac{dy}{dx} + 3y = 0$

$2x + \frac{dy}{dx} + 3x \frac{dy}{dx} = -2x - 3y$

$dy/dx (2y + 3x) = -2x - 3y$

$dy/dx = \frac{-2x - 3y}{2y + 3x}$

$m = dy/dx \,|\, x=1, y=2 = \frac{-2(1) - 3(2)}{2(2) + 3(1)}$

$= \frac{-2 - 6}{4 + 3} = \frac{-8}{7}$

(a) Equation of tangent

$y - y_1 = m(x - x_1)$

$y - 2 = \frac{-8}{7}(x - 1)$

$7(y - 2) = -8(x - 1)$

$7y - 14 = -8x + 8$

$7y + 8x - 14 - 8 = 0$

$7y + 8x - 22 = 0$ is the equation of tangent.

b) Equation of normal

$y - y_1 = \frac{-1}{m}(x - x_1)$

$y - 2 = \frac{-1}{-8/7}(x - 1)$

$y - 2 = 7/8 (x - 1)$

$8(y - 2) = 7(x - 1)$

$8y - 16 = 7x - 7$

$8y - 7x - 16 + 7 = 0$

$8y - 7x - 9 = 0$ is the equation of normal.

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