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19/MHS11/095 PHARMACY MAT 104

1) $y = \frac{1}{(x-2)}$ defined

The function is defined for all real numbers except $x=2$

Domain = Set of Real numbers except $x=2$

Codomain = Set of Real numbers except $y=0$

2) If $K = \ln v$, differentiate K

$$\frac{d}{dK}(\ln v) = \frac{1}{v}\frac{dv}{dK}$$

3) Express y as an explicit function of x in the following

a $2x - 3y + 2 = 0$

Make y the subject of the formular

$2x + 2 = 3y$

$$y = \frac{2x+2}{3}$$

b) $x^2 + y^2 = 4$

$y^2 = 4 - x^2$

$y = \pm\sqrt{4 - x^2}$

4) If $P = \sin^{-1} t$, find the derivative of P

$$P = \frac{t}{\sin}$$

$t = \sin P$ — ①

differentiating both sides

$$\frac{dt}{dp} = \cos p$$

but we want dp/dt, therefore

$$\frac{dp}{dt} = \frac{1}{\cos p}$$

Recall that $\cos^2 p + \sin^2 p = 1$

$\cos^2 P = 1 - \sin^2 p$

$\cos P = \sqrt{1 - \sin^2 p}$

$\cos P = \sqrt{1 - \sin^2 p}$

but $\sin P = t \Rightarrow \sin^2 P = t^2$

$\therefore \cos P = \sqrt{1-t^2}$

Hence

$$\frac{dP}{dt} = \frac{1}{\cos P} = \frac{1}{\sqrt{1-t^2}}$$

5) If $f(x) = 2x^2-5$ and $g(x) = 4x-2$, find $f\circ g(x)$ and $g\circ f(x)$

a) $(f\circ g)x = f(g(x))$

$f(x) = 2x^2-5$

$g(x) = 4x-2$

$$\begin{aligned} f(g(x)) &= f(4x-2) \\ &= 2(4x-2)^2-5 \\ &= 2(4x-2)(4x-2)-5 \\ &= 2(16x^2-8x-8x+4)-5 \\ &= 32x^2-32x+8-5 \\ &= 32x^2-32x+3 \end{aligned}$$

b) $(g\circ f)x = g(f(x))$

$$\begin{aligned} &= g(2x^2-5) \\ &= 4(2x^2-5)-2 \\ &= 8x^2-20-2 \\ &= 8x^2-22 \end{aligned}$$

6) If $f(x) = 3x^2-2x+1 = 0$, show that $f_e(x) + f_o(x) = f(x)$

$$f_e(x) = \frac{f(x+f(-x)}{2}$$

$f(-x) = 3(-x)^2-2(-x)+1$

$f(-x) = 3x^2+2x+1$

$$f_e(x) = \frac{(3x^2-2x+1)+(3x^2+2x+1)}{2}$$

$$f_e(x) = \frac{6x^2+2}{2}$$

$$= \frac{\cancel{2}(3x^2+1)}{\cancel{2}} = 3x^2+1$$

$$f_o(x) = \frac{f(x)-f(-x)}{2}$$

$$= \frac{(3x^2-2x+1)-(3x^2+2x+1)}{2}$$

$$= \frac{-4x}{2} = -2x$$

$f(x) = fe(x) + fo(x)$
$= (3x^2+1) + (-2x)$
$3x^2 + 1 - 2x$
$\therefore f(x) = fe(x) + fo(x)$

7) Differentiate $y = \cos x$ from first principle
$y = \cos x$
$y + \delta y = \cos(x + \delta x)$
Make δy the subject of the formular by subtracting y from both sides
$\delta y = \cos(x+\delta x) - y$
But $y = \cos x$
$\delta y = \cos(x+\delta x) - \cos x$ — (*)
Consider from Trig
$\cos(A+B) = \cos A \cos B - \sin A \sin B$
$- \cos(A-B) = \cos A \cos B + \sin A \sin B$
$\cos(A+B) - \cos(A-B) = -2\sin A \sin B$ — (2*)
Compare (*) and (2*) : Let
$A + B = x + \delta x$ — (i)
$A - B = x$ — (ii)
Add (i) and (ii)
$2A = 2x + \delta x$
$A = \frac{2x}{2} + \frac{\delta x}{2}$
$A = x + \delta x/2$
Sub $A = x + \delta x/2$ in Equ (ii)
$x + \frac{\delta x}{2} - B = x$
$x + \delta x/2 - x = B$
$\delta x/2 = B$ } 3*
$A = x + \delta x/2$
Compare (*) and 2*
$\cos(x + \delta x) - \cos x = -2\sin A \sin B$
$\cos(x+\delta x) - \cos x = -2\sin\left(x + \frac{\delta x}{2}\right)\left(\sin\frac{\delta x}{2}\right)$

$\delta y = -2\sin(x + \delta x/2)\sin(\delta x/2)$
$\frac{\delta y}{\delta x} = \frac{-2\sin(x + \delta x/2)\sin(\delta x/2)}{\delta x}$

$$\frac{\delta y}{\delta x} = -\frac{\sin(x+\delta x/2)\sin(\delta x/2)}{\delta x/2}$$

$$\frac{\delta y}{\delta x} = -\sin(x+\delta x/2)\frac{\sin(\delta x/2)}{\delta x/2}$$

A standard limit

$$\lim_{\delta x \to 0} \frac{\sin(\delta x/2)}{\delta x/2} = 1$$

Find limit of (4) as $\delta x \to 0$

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = -\sin(x+\delta x/2)\frac{\sin(\delta x/2)}{\delta x/2}$$

$$= -\sin(x+0)\cdot 1 = -\sin x$$

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \frac{dy}{dx} = -\sin x$$

8) Find dy/dx if $y = 3t^2$ and $x = 1/t^2$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= \frac{dy}{dt} \div \frac{dx}{dt}$$

$$\frac{dy}{dt} = 6t \; ; \; \frac{dx}{dt} = \frac{-2}{t^3}$$

$$\frac{dy}{dx} = 6t \div \frac{-2}{t^3}, \; 6\cancel{t} \times \frac{-2}{t^{\cancel{3}}}$$

$$= 6 \times \frac{-2}{t^2} = \frac{-12}{t^2}$$

$$\therefore dy/dx = \frac{-12}{t^2}$$

9) $y = x^2 \cos 2x \, e^{4x}$

Taking $\log_e$ of both sides

$$\ln y = \ln x^2 + \ln \cos 2x + \ln e^{4x}$$

Differentiating both wrt x

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x^2}(2x) + \frac{1}{\cos 2x}(-2\sin 2x) + 4$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x} - \frac{2\sin 2x}{\cos 2x} + 4$$

Multiplying both sides by y

$$\frac{dy}{dx} = y\left(\frac{2}{x} - \frac{2\sin 2x}{\cos 2x} + 4\right)$$

$$= x^2 \cos 2x \, e^{4x} \times \frac{2}{x} - \frac{2\sin x}{\cos x} + 4$$

10) $y = \sin(3x^3 + 5)$

Let $u = 3x^3 + 5$

$$\frac{dy}{dx} = \cos u$$

$$\frac{du}{dx} = 9x^2$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= \cos u \times 9x^2$$

$$= 9x^2 \cos u$$

$$= 9x^2 \cos 3x^3 + 5$$