

UNIVERSITY OF CALIFORNIA
19/11/2014/415
MAT 104 ASSIGNMENT 1.

Examine whether or not these pair of lines are perpendicular to each other.

1) $y - 3x - 2 = 0$ and $3y + x + 9 = 0$.

Sol.

For 2 lines to be perpendicular to each other, $m_1 m_2 = -1$

$$y - 3x - 2 = 0$$

$$y = 3x + 2$$

$$y = mx + c$$

$$\text{So } m_1 = 3$$

$$\text{Also, } 3y + x + 9 = 0$$

$$3y = -x - 9$$

$$y = -\frac{x}{3} - \frac{9}{3}$$

$$y \neq m_2 = -\frac{1}{3}$$

$$m_1 m_2 = 3 \times -\frac{1}{3} = -1$$

Since $m_1 m_2 = -1$, the pair of lines $y - 3x - 2 = 0$ and $3y + x + 9 = 0$ are said to be perpendicular to each other.

2) $3y - 4 = 2x + 3$ and $y - 5 = x + 6$ ✓

Sol

$$3y = 2x + 3 + 4$$

$$3y = 2x + 7$$

$$y = \frac{2x + 7}{3}$$

$$y - 5 = x + 6$$

$$y = x + 6 + 5$$

$$y = x + 11$$

$$m_1 = \frac{2}{3}$$

$$m_2 = 1$$

$$\therefore m_1 m_2 = -1$$

$$\frac{2}{3} \cdot 1 = 1 \quad (\neq -1)$$

They are not perpendicular to each other.

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3) Find the equations of the tangent and normal to the curve $x^2 + y^2 + 3xy - 11 = 0$ at the point $x = 1, y = 2$.

sol

$$2x + 2y \frac{dy}{dx} + 3(x \frac{dy}{dx} + y \cdot 1) - 0 = 0$$

$$2x + 2y \frac{dy}{dx} + 3x \frac{dy}{dx} + 3y = 0$$

$$2x + \frac{dy}{dx} (2y + 3x) + 3y = 0$$

$$\frac{dy}{dx} \frac{(2y + 3x)}{2y + 3x} = \frac{-2x - 3y}{2y + 3x}$$

$$\frac{dy}{dx} = \frac{-2x - 3y}{2y + 3x}$$

$$m_1 = \frac{dy}{dx} = \frac{-2(1) - 3(2)}{2(2) + 3(1)} = \frac{-8}{7}$$

$$m_1 = -\frac{8}{7}$$

Eqn of tangent, $M_1 = m_1$

$$y - y_1 = m_1(x - x_1)$$

$$y - 2 = -\frac{8}{7}(x - 1)$$

$$7y - 14 = -8x - 8$$

$$7y + 8x - 14 - 8 = 0$$

$7y + 8x - 22 = 0$ → Equation of the tangent

Eqn of normal $m_2 = \frac{1}{m_1}$

$$y - y_1 = \frac{1}{m_1}(x - x_1)$$

$$y - 2 = -\frac{1}{-\frac{8}{7}}(x - 1)$$

$$y - 2 = \frac{7}{8}(x - 1)$$

$$8y - 16 = 7x - 7$$

$$8y - 7x - 16 + 7 = 0$$

$8y - 7x - 9 = 0$ → Equation of the normal.