3.) Find the equations of the tangent and normal to the curve $x^2 + y^2 + 3xy - 11 = 0$ at the point $x=1$, $y=2$.

Solution

$x^2 + y^2 + 3xy - 11 = 0$

$x_1 = 1$ and $y_1 = 2$.

$2x + 2y\frac{dy}{dx} + 3\left(x\frac{dy}{dx} + y\right) = 0$

$2x + 2y\frac{dy}{dx} + 3x\frac{dy}{dx} + 3y = 0$

$2x + 3y + 2y\frac{dy}{dx} + 3x\frac{dy}{dx} = 0$

$2y\frac{dy}{dx} + 3x\frac{dy}{dx} = -2x - 3y$.

$\frac{dy}{dx}(2y + 3x) = -2x - 3y$.

$\frac{dy}{dx} = \frac{-2x - 3y}{2y + 3x}$.

$m = \frac{dy}{dx}$ at $x=1$ and $y=2$. $= \frac{-2(1) - 3(2)}{2(2) + 3(1)} = \frac{-2-6}{4+3} = -\frac{8}{7}$.

To find the equation of the tangent,

$y - y_1 = m(x - x_1)$.

$y - 2 = -\frac{8}{7}(x - 1)$.

$7(y-2) = -8(x-1)$.

$7y - 14 = -8x + 8$.

$7y + 8x = 8 + 14$

$7y + 8x = 22$.

$7y + 8x - 22 = 0$

To find the equation of the normal;

$m_1 m_2 = -1$.

$-\frac{8}{7} m_2 = -1$.

$m_2 = -1 \times -\frac{7}{8}$.

$m_2 = \frac{7}{8}$.

$y - y_1 = m(x - x_1)$.

$y - 2 = \frac{7}{8}(x - 1)$.

$8(y-2) = 7(x-1)$.

$8y - 16 = 7x - 7$

$8y - 7x = -7 + 16$.

$8y - 7x = +9$.

$8y - 7x - 9 = 0$

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Assignment IX:

Examine whether or not these pair of lines are perpendicular to each other.

(1) $y - 3x - 2 = 0$ and $3y + x + 9 = 0$

(2) $3y - 4 = 2x + 3$ and $y - 5 = x + 6$

Solution.

1.) $y - 3x - 2 = 0$ and $3y + x + 9 = 0$

Using: $y = mx + c$; m = intercept.

From equation 1: $y - 3x - 2 = 0$

$y = 3x + 2$.

$\therefore m_1 = 3$.

From equation 2: $3y + x + 9 = 0$

$3y = -x - 9$.

$y = -\frac{x}{3} - \frac{9}{3}$

$y = -\frac{x}{3} - 3$.

$\therefore m_2 = -\frac{1}{3}$.

If the two lines are perpendicular, $m_1 \times m_2 = -1$

Hence, $3 \times -\frac{1}{3} = -1$. $\therefore$ The pair of lines are perpendicular.

2.) $3y - 4 = 2x + 3$ and $y - 5 = x + 6$

Using: $y = mx + c$; m = intercept.

From equation 1: $3y - 4 = 2x + 3$.

$3y = 2x + 3 + 4$.

$3y = 2x + 7$.

$y = \frac{2}{3}x + \frac{7}{3}$.

$m_1 = \frac{2}{3}$.

From equation 2: $y - 5 = x + 6$.

$y = x + 6 + 5$.

$y = x + 11$.

$m_2 = 1$.

If the two lines are perpendicular, $m_1 \times m_2 = -1$.

Hence, $\frac{2}{3} \times 1 = \frac{2}{3}$. $\therefore$ The pair of lines are not perpendicular.