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Determine whether or not these pair of lines are perpendicular to each other.

1)  $y - 3x - 2 = 0$  and  $3y + 2x + 9 = 0$

2)  $3y - 4 = 2x + 3$  and  $y - 5 = x + 6$

3) Find the equations of the tangent and normal to the arc  $x^2 + y^2 + 6xy - 4 = 0$  at the point  $x = \frac{1}{2}, y = 2$ .

### Solutions

1)  $y - 3x - 2 = 0$

$3y + 2x + 9 = 0$

For the lines to be perpendicular then  $md_1 = -1$ .

$y = 3x - 2 = 0$

Making  $y$  the subject of the formula

$y = 3x + 2$

By comparison  $y = mx + c$

$md_1 = 3$

$3y + 2x + 9 = 0$

Making  $y$  the subject of the formula

$y = \frac{3y}{3} = \frac{-2x - 9}{3}$

$y = \frac{-2x}{3} - 3$

$y = mx + c, md_2 = -\frac{2}{3}$

$md_1 = -1$ .

$3x - \frac{y}{3} = -1$  since  $md_1 = -1$  then the lines  $y - 3x - 2$

and  $3y + 2x + 9 = 0$  ARE PERPENDICULAR.

$$2) 3y - 4 = 2x + 5 \quad \text{--- (1)}$$

$$y - 5 = 2x + 6 \quad \text{--- (2)}$$

making y the subject of the formula

$$3y = 2x + 5 + 4$$

$$\frac{3y}{3} = \frac{2x}{3} + \frac{9}{3}$$

$$y = \frac{2x}{3} + 3, \text{ but } y = 2x + 6$$

$$\therefore m_1 = \frac{2}{3}$$

$$y - 5 = 2x + 6$$

making y the subject formula,

$$y = 2x + 6 + 5$$

$$y = 2x + 11$$

By comparison  $y = mx + c$ ,

$$m_2 = 2$$

for lines to be perpendicular,

$$m_1 m_2 = -1$$

$$\frac{2}{3} \times 2 = -\frac{2}{3}$$

$$m_1 m_2 \neq -1$$

hence the lines  $3y - 4 = 2x + 5$  and  $y - 5 = 2x + 6$  are

NOT PERPENDICULAR

$$3) u^2 + y^2 + 3xy - 11 = 0 \quad (u=1, y=2)$$

$$m = \frac{dy}{dx}$$

$$u^2 + y^2 + 3xy - 11 = 0$$

$$\frac{dy}{dx}$$

$$2u + 2y \frac{dy}{dx} + 3(u \frac{dy}{dx} + y) = 0$$

$$2u + 2y \frac{dy}{dx} + 3(u \frac{dy}{dx} + y) = 0$$

$$2u + 2y \frac{dy}{dx} + 3u \frac{dy}{dx} = -2x - 3y$$

$$\frac{dy}{dx} (2y + 3u) = -2u - 3y$$

$$\frac{dy}{dx} = \frac{-2u - 3y}{2y + 3u}$$

$$m = \frac{dy}{dx} \Big|_{(u=1, y=2)} = \frac{-2(1) - 3(2)}{2(2) + 3(1)} = \frac{-2 - 6}{4 + 3} = \frac{-8}{7}$$

a) Equation of tangent

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \frac{-8}{7}(x - 1)$$

$$7(y - 2) = -8(x - 1)$$

$$7y - 14 = -8x + 8$$

$$7y + 8x - 14 - 8 = 0$$

$7y + 8x - 22 = 0$ , is the equation of tangent.

b) Equation of normal

$$y - y_1 = -1/m(x - x_1)$$

$$y - 2 = -1/(-8/7)(x - 1)$$

$$y - 2 = 7/8(x - 1)$$

$$8(y - 2) = 7(x - 1)$$

$$8y - 16 = 7x - 7$$

$$8y - 7x - 16 + 7 = 0$$

$8y - 7x - 9 = 0$  is the equation of normal.