

ORAKPO MIRABEL

18/SCI01/074

1. (i) Linear combination of vectors

This is a mathematical method of combining vectors using addition and scalar multiplication

i.e. $\alpha_1v_1 + \alpha_2v_2 + \alpha_3v_3 + \dots + \alpha_nv_n$

(ii) Linear dependence of vectors

This occurs when the scalars in a linear combination are not all equal to zero

i.e. $\alpha_1u_1 + \alpha_2u_2 + \alpha_3u_3 + \dots + \alpha_nu_n = 0$

2. $U\alpha + V\beta + W\gamma = (a, b, c)$

$$\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \alpha + \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \beta + \begin{bmatrix} 1 \\ 1 \\ -4 \end{bmatrix} \gamma = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\alpha + 2\beta + \gamma = a \dots\dots(i)$$

$$\beta + \gamma = b \dots\dots(ii)$$

$$-\alpha + 3\beta - 4\gamma = c \dots\dots(iii)$$

From equ (ii)

$$\beta = b - \gamma \dots\dots(iv)$$

Put equ (iv) into (i) and (iii)

$$\alpha + 2(b - \gamma) + \gamma = a$$

$$\alpha + 2b - 2\gamma + \gamma = a$$

$$\alpha + 2b - \gamma = a$$

$$\alpha - \gamma = a - 2b \dots\dots(v)$$

For equ (iii)

$$-\alpha + 3(b - \gamma) - 4\gamma = c$$

$$-\alpha + 3b - 3\gamma - 4\gamma = c$$

$$-\alpha + 3b - 7\gamma = c$$

$$-\alpha - 7\gamma = c - 3b \dots\dots(vi)$$

Compare equ (v) and (vi) by addition

$$\alpha - \gamma = a - 2b$$

$$-\alpha - 7\gamma = c - 3b$$

$$\hline -8\gamma = a - 2b + c - 3b$$

$$-8\gamma = a - 5b + c$$

$$\gamma = \frac{(a - 5b + c)}{-8}$$

$$\gamma = -\frac{(a - 5b + c)}{8}$$

$$\gamma = \frac{-a + 5b - c}{8}$$

Put γ in (ii)

$$\beta + \left(\frac{-a + 5b - c}{8} \right) c = b$$

$$\beta = \frac{b - (-a + 5b - c)}{8}$$

$$\beta = \frac{b + a - 5b + c}{8}$$

$$\beta = \frac{8b + a - 5b + c}{8}$$

$$\beta = \frac{a + 3b + c}{8}$$

Put β and γ into equ (i)

$$\alpha + 2\left(\frac{a + 3b + c}{8}\right)c - \left(\frac{-a + 5b}{8}\right)c = a$$

$$\alpha + \left(\frac{a + 3b + c}{4}\right)c - \left(\frac{-a + 5b}{8}\right)c = a$$

$$\alpha = a - \left(\frac{a + 3b + c}{4}\right)c + \left(\frac{-a + 5b}{8}\right)c$$

$$\alpha = a - \frac{a - 3b - c}{4} + \frac{a - 5b + c}{8}$$

$$\alpha = \frac{8a + 2(-a - 3b - c) + a - 5b + c}{8}$$

$$\alpha = \frac{8a - 2a - 6b - 2c + a - 5b + c}{8}$$

$$\alpha = \frac{7a - 11b - c}{8}$$

$$\left(\frac{7a - 11b - c}{8} \right) U + \left(\frac{a + 3b + c}{8} \right) V + \left(\frac{-a + 5b - c}{8} \right) W$$

3. (a) Commutativity of vector addition
(b) Associativity of vector addition
(c) Identity element of addition
(d) Inverse element of addition