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COURSE : MAT 104 (GENERAL MATHEMATICS III)

1 Examine whether or not these pair of lines are perpendicular to each other.

$y - 3x - 2 = 0$ and $3y + x + 9 = 0$

Solution

The equation of a straight line is expressed as $y = mx + c$, has gradient "m", $y = m_1x + c_1$ and $y = m_2x + c_2$ are perpendicular

$y - 3x - 2 = 0$

make y the subject of the formula

$y = 3x + 2$

Recall, $y = mx + c$

$m_1 = 3$

$3y + x + 9 = 0$

make y the subject of the formula

$3y + x + 9 = 0$

$3y = -x - 9$

Divide through by 3

$$\frac{\cancel{3}y}{\cancel{3}} = \frac{-x}{3} - \frac{\cancel{9}\,3}{\cancel{3}\,1}$$

$$y = -\frac{x}{3} - 3$$

$$y = \frac{-1}{3}x - 3$$

Recall, $y = mx + c$

$$m_2 = \frac{-1}{3}$$

$m_1 m_2 = -1$ (for Perpendicularity)

$$m_1 \times m_2 = {}^1\cancel{3} \times \frac{-1}{\cancel{3}_1}$$

$m_1 m_2 = -1$

$\therefore$ The lines $y - 3x - 2 = 0$ and $3y + x + 9 = 0$ are Perpendicular

2. $3y - 4 = 2x + 3$ and $y - 5 = x + 6$

Solution

The equation of a straight line is expressed as $y = mx + c$, has gradient "m". $y = m_1 x + c_1$ and $y = m_2 x + c_2$ are perpendicular if $m_1 m_2 = -1$

$3y - 4 = 2x + 3$ ---- equ 1

make y the subject of the formula

$3y = 2x + 3 + 4$

Divide through by 3

$$\frac{\cancel{3}y}{\cancel{3}} = \frac{2x}{3} + \frac{\cancel{3}}{\cancel{3}} + \frac{4}{3}$$

$$y = \frac{2x}{3} + \frac{4}{3}$$

Recall $y = mx + c$

$\therefore m_1 = \frac{2}{3}$

$y - 5 = x + 6$

make y the Subject of the formula

$y = x + 6 + 5$

$y = x + 11$

Recall, $y = mx + c$

$\therefore m_2 = 1$

$m_1 m_2 = -1$ (for Perpendicularity)

$m_1 \times m_2 = \frac{2}{3} \times 1$

$m_1 m_2 = \frac{2}{3}$

$\therefore$ the lines $3y-4 = 2x+3$ and $y-5 = x+3$ are not Perpendicular

3 Find the equation of the tangent and normal to the curve $x^2+y^2+3xy-11=0$ at the Point $x=1, y=2$

Solution

$x^2+y^2+3xy-11=0$ $(x=1, y=2)$

$m = \frac{dy}{dx}$

$x^2+y^2+3xy-11=0$

Differentiating by implicit method

$2x + 2y\frac{dy}{dx} + 3\left(x\frac{dy}{dx} + y\right) = 0$

$2x + 2y\frac{dy}{dx} + 3\left(x\frac{dy}{dx} + y\right) = 0$

$2x + 2y\frac{dy}{dx} + 3x\frac{dy}{dx} + 3y = 0$

$2y\frac{dy}{dx} + 3x\frac{dy}{dx} = -2x-3y$

$\frac{dy}{dx}\left(2y+3x\right) = -2x-3y$

$\frac{dy}{dx} = \frac{-2x-3y}{2y+3x}$

a. for the equation of the tangent

$\frac{dy}{dx} = \frac{-2x-3y}{2y+3x}$

$m_1 = \frac{dy}{dx}\Big|_{x=x_1}$

$\frac{dy}{dx} = \frac{-2(1)-3(2)}{2(2)+3(1)} = \frac{-2-6}{4+3}$

$\frac{dy}{dx} = \frac{-8}{7}$

$y - y_1 = m(x - x_1)$

$y - 2 = \frac{-8}{7}(x - 1)$

Cross multiply

$\frac{y-2}{1} = \frac{-8(x-1)}{7}$

$7(y-2) = -8(x-1)$

$7y - 14 = -8x - 8$

$7y + 8x = 8 + 14$

$7y + 8x = 22$

$7y + 8x - 22 = 0$

$\therefore$ The equation of the tangent is $7y + 8x - 22 = 0$

b. For equation of the normal

$m_2 = \frac{-1}{m_1} = \frac{-1}{1} \times \frac{7}{-8} = \frac{7}{8}$

$\therefore m_2 = \frac{7}{8}$

$y - y_1 = m_2(x_2 - x_1)$

$y - 2 = \frac{7}{8}(x - 1)$

Cross multiply

$\frac{y-2}{1} = \frac{7(x-1)}{8}$

$8(y-2) = 7(x-1)$

$8y - 16 = 7x - 7$

$8y - 7x = -7 + 16$

$8y - 7x = 9$

$8y - 7x - 9 = 0$

$\therefore$ The equation of the normal is $8y - 7x - 9 = 0$