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MATRIC NUMBER: 19/MHS01/320
COURSE: MAT 104

ASSIGNMENT

1 Examine whether or not these pair of lines are perpendicular to each other.

1. $y-3x-2=0$ and $3y+x+9=0$

Solution

The equation of a straight line is expressed as $y=mx+c$, has gradient "m", $y=m_1x+c_1$ and $y=m_2x+c_2$ are perpendicular if $m_1m_2=-1$

$y-3x-2=0$

Make y the subject of the formular.

$y=3x+2$

Recall, $y=mx+c$

$m_1=3$

$3y+x+9=0$

Make y the subject of the formular.

$3y+x+9=0$

$3y=-x-9$

Divide through by 3

$\frac{\cancel{3}y}{\cancel{3}} = \frac{-x}{3} - \frac{\cancel{9}^{3}}{\cancel{3}_{1}}$

$y = -\frac{x}{3} - 3$

$y = \frac{-1x}{3} - 3$

Recall, $y = mx + c$

$$m_2 = \frac{-1}{3}$$

$m_1 m_2 = -1$ (for perpendicularity)

$$m_1 \times m_2 = \cancel{3}^{1} \times \frac{-1}{\cancel{3}_1}$$

$$m_1 m_2 = -1$$

$\therefore$ The lines $y - 3x - 2 = 0$ and $3y + x + 9 = 0$ are perpendicular

2. $3y - 4 = 2x + 3$ and $y - 5 = x + 6$

Solution

The equation of a straight line is expressed as $y = mx + c$, has gradient 'm'. $y = m_1 x + c_1$ and $y = m_2 x + c_2$ are perpendicular if $m_1 m_2 = -1$

$3y - 4 = 2x + 3$ - - - - - - - - eqn 1

Make y the subject of the formular.

$3y = 2x + 3 + 4$

Divide through by 3

$$\frac{\cancel{3}y}{\cancel{3}} = \frac{2x}{3} + \frac{\cancel{3}}{\cancel{3}} + \frac{4}{3}$$

$$y = \frac{2x}{3} + \frac{4}{3}$$

Recall, $y = mx + c$

$$\therefore m_1 = \frac{2}{3}$$

$y - 5 = x + 6$

Make y the subject of the formular

$y = x + 6 + 5$

$y = x + 11$

Recall, $y = mx + c$

$\therefore m_2 = 1$

$m_1 m_2 = -1$ (for perpendicularity)

$m_1 \times m_2 = \frac{2}{3} \times 1$

$m_1 m_2 = \frac{2}{3}$

$\therefore$ The lines $3y - 4 = 2x + 3$ and $y - 5 = x + 3$ are not perpendicular.

Find the equations of the tangent and normal to the curve $x^2 + y^2 + 3xy - 11 = 0$ at the point $x = 1$, $y = 2$.

Solution

$x^2 + y^2 + 3xy - 11 = 0 \quad (x = 1, y = 2)$

$m = \frac{dy}{dx}$

$x^2 + y^2 + 3xy - 11 = 0$

<u>Differentiating by Implicit method</u>

$$2x + 2y\frac{dy}{dx} + 3\left(x \cdot \frac{dy}{dx} + y\right) = 0$$

$$2x + 2y\frac{dy}{dx} + 3\left(x \cdot \frac{dy}{dx} + y\right) = 0$$

$$2x + 2y\frac{dy}{dx} + 3x\frac{dy}{dx} + 3y = 0$$

$$2y\frac{dy}{dx} + 3x\frac{dy}{dx} = -2x - 3y$$

$$\frac{dy}{dx}(2y + 3x) = -2x - 3y$$

$$\frac{dy}{dx} = \frac{-2x - 3y}{2y + 3x}$$

a) For the equation of the tangent.

$$\frac{dy}{dx} = \frac{-2x - 3y}{2y + 3x}$$

$$m_1 = \frac{dy}{dx}\Big|_{x = x_1}$$

$$\frac{dy}{dx} = \frac{-2(1) - 3(2)}{2(2) + 3(1)} = \frac{-2-6}{4+3}$$

$$\frac{dy}{dx} = \frac{-8}{7}$$

$$y - y_1 = m(x - x_1)$$
$$y - 2 = \frac{-8}{7}(x - 1)$$

Cross multiply

$$\frac{y-2}{1} = \frac{-8(x-1)}{7}$$

$$7(y-2) = -8(x-1)$$

$7y - 14 = -8x + 8$

~~The equation for tangent is $7y +$~~

$7y + 8x = 8 + 14$

$7y + 8x = 22$

$7y + 8x - 22 = 0$

$\therefore$ The equation for tangent is $7y + 8x - 22 = 0$

b For equation of the normal.

$$m_2 = \frac{-1}{m_1} = \frac{-1}{1} \times \frac{7}{-8} = \frac{7}{8}$$

$$\therefore m_2 = \frac{7}{8}$$

$y - y_1 = m_2(x_2 - x_1)$

$y - 2 = \frac{7}{8}(x - 1)$

Cross multiply

$$\frac{y-2}{1} = \frac{7}{8}(x-1)$$

$8(y-2) = 7(x-1)$

$8y - 16 = 7x - 7$

$8y - 7x = -7 + 16$

$8y - 7x = 9$

$8y - 7x - 9 = 0$

$\therefore$ The equation of the normal is $8y - 7x - 9 = 0$.