NAME - Eweiye Ruth Ehinome
DEPARTMENT - Medical Laboratory Science
MATRIC NO - 19/MHS06/014
COURSE CODE - MAT 104

1] The function is defined at all real numbers except $x = 2$
Domain is the set of real numbers except $x = 2$
Codomain is the set of real numbers except $y = 0$

2] $K = \ln V$

$$\frac{d}{dK}[\ln V] = \frac{1}{V}$$

3a] $2x - 3y - 2 = 0$

$2x - 2 = 3y$

$$y = \frac{2x - 2}{3}$$

b] $x^2 + y^2 = 4$

$y^2 = 4 - x^2$

$y = \sqrt{4 - x^2}$

4] $P = \sin^{-1} t$

$P = \frac{t}{\sin}$

$t = \sin P$ --- (1)

Recall that, $\sin^2 P + \cos^2 P = 1$ --- (2)

$\frac{dt}{dP}$ of $t = \cos P$

From equa. (2)

$\sin^2 P + \cos^2 P = 1$

$\cos^2 P = 1 - \sin^2 P$

$\cos P = \sqrt{1 - \sin^2 P}$

From equa. (1)

$t = \sin P$

$\cos P = \sqrt{1 - t^2}$

$\frac{dt}{dP} = \cos P = \sqrt{1 - t^2}$

$\therefore \frac{dP}{dt} = \frac{1}{\sqrt{1 - t^2}}$

5] $f[x] = 2x^2 - 5 \qquad g[x] = 4x - 2$

$fog[x] = f[g[x]]$

$f[x] = 2x^2 - 5$

$g[x] = 4x - 2$

$f[g[x]] = f[4x - 2]$

$= 2[4x - 2]^2 - 5$

$= 2[4x - 2][4x - 2] - 5$

$fog[x] = 2[16x^2 - 16x + 4] - 5$

$= 32x^2 - 32x + 8 - 5$

$= 32x^2 - 32x + 3$

$gof[x] = g[f[x]]$

$= g[2x^2 - 5]$

$= 4[2x^2 - 5] - 2$

$= 8x^2 - 20 - 2$

$= 8x^2 - 22$

6) $$f_e[x] = \frac{f[x] + f[-x]}{2}$$

$$f[-x] = 3[-x]^2 - 2[-x] + 1$$

$$f[-x] = 3x^2 + 2x + 1$$

$$f_e[x] = \frac{3x^2 - 2x + 1 + 3x^2 + 2x + 1}{2}$$

$$f_e[x] = \frac{6x^2 + 2}{2} = \frac{\cancel{2}[3x^2 + 1]}{\cancel{2}}$$

$$f_e[x] = 3x^2 + 1$$

$$f_o[x] = \frac{f[x] - f[-x]}{2}$$

$$f_o[x] = \frac{\cancel{3x^2} - 2x \cancel{+1} \cancel{-3x^2} - 2x \cancel{-1}}{2}$$

$$f_o[x] = \frac{-4x}{\cancel{2}} = -2x$$

$$f_e[x] + f_o[x] = f[x]$$

$$f[x] = 3x^2 + 1 - 2x$$

$$f[x] = 3x^2 - 2x + 1$$

7] $y = \cos x$

$y + \delta y = \cos[x + \delta x]$

Subtract y from both sides

$\delta y = \cos[x + \delta x] - y$

but $y = \cos x$

$\delta y = \cos[x + \delta x] - \cos x$ --- (1)

Consider from trig.

$\cos[A+B] = \cos A \cos B - \sin A \sin B$

$\cos[A-B] = \cos A \cos B + \sin A \sin B$

$\cos[A+B] - \cos[A-B] = -2\sin A \sin B$ --- (2)

Compare (1) and (2)

Let

$A+B = x + \delta x$ --- (i)

$A+B = x$ --- (ii)

Add equa (i) and (ii)

$2A = 2x + \delta x$

$A = \frac{2x + \delta x}{2}$

$A = x + \frac{\delta x}{2}$ $\quad B = \frac{\delta x}{2}$ --- (3)

Compare equa. (1) and (2)

$\cos[x + \delta x] - \cos x = -2\sin A \sin B$

$\cos[x + \delta x] - \cos x = -2\sin[x + \delta x/2] \sin[\delta x/2]$

$\delta y = 2\sin[x + \delta y/2] \sin[\delta x/2]$

$$\frac{\delta y}{\delta x} = \frac{-2\sin[x+\delta x/2]\sin[\delta x/2]}{\delta x}$$

$$\frac{\delta y}{\delta x} = \frac{-\sin[x+\delta x/2]\sin[\delta x/2]}{\delta x/2}$$

$$\frac{\delta y}{\delta x} = -\sin[x+\delta x/2]\frac{\sin[\delta x/2]}{\delta x/2} \quad --- (4)$$

Standard limit

$$\lim_{\delta x \to 0} \frac{\sin[\delta x/2]}{\delta x/2} = 1$$

limit of (4) as $\delta x \to 0$

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \lim_{\delta x \to 0} -\sin[x+\delta x/2]\frac{\sin[\delta x/2]}{\delta x/2}$$

$$= -\sin[x+0]$$

$$= -\sin x$$

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \frac{dy}{dx}$$

$$= -\sin x$$

8] $y = 3t^2$ and $x = 1/t^2$

$y = 3t^2$; $\frac{dy}{dx} = 6t$

$x = \frac{1}{t^2}$; $\frac{dy}{dx} = -1/2t = -2t$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$\frac{dy}{dx} = \frac{6t}{-2t}$$

$$\frac{dy}{dx} = -3t$$

9] $y = x^2 \cos 2x e^{4x}$

$y = x^2 \cos[2xe^{4x}]$

Using the product rule

$$\frac{dy}{dx} = v\frac{du}{dx} + u\frac{dv}{dx}$$

Let $u = x^2$ and $v = \cos 2xe^{4x}$

$$\frac{du}{dx} = 2x$$

$v = \cos[2xe^{4x}]$

Using the chain rule

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$u = 2xe^{4x}$ and $y = \cos u$

$\frac{dy}{dx} = 2\times 4\times e^{4x}$ and $y = -\sin u$

$= 8e^{4x}$ and $y = -\sin u$

$dy/dx = 8e^{4x} \times -\sin[2xe^{4x}]$

$= -8e^{4x}\sin[2xe^{4x}]$

$$\frac{dy}{dx} = \cos[2xe^{4x}] \times 2x + x^2 \times -8e^{4x}\sin[2xe^{4x}]$$

$$= 2x\cos[2xe^{4x}] + -8x^2e^{4x}\sin[2xe^{4x}]$$

10] $y = \sin[3x^3 + 5]$

Let $u = 3x^3 + 5$ $\quad \frac{du}{dx} = 9x^2$

$y = \sin u$ $\quad \frac{dy}{du} = \cos u$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{dy}{dx}$$

$$= \cos u \times 9x^2$$

$$= 9x^2 \cos u$$

but $u = 3x^3 + 5$

$$\frac{dy}{dx} = 9x^2 \cos[3x^3 + 5]$$