MATRIC NO: 19/MHS01/196

S/N: 002 DEPT: MEDICINE AND SURGERY

MAT 104

1. Examine whether or not these pair of lines are perpendicular to each other

i $y - 3x - 2 = 0$ and $3y + x + 9 = 0$

ii $3y - 4 = 2x + 3$ and $y - 5 = x + 6$

3 Find the equations of the tangent and normal to the curve, $x^2 + y^2 + 3xy - 11 = 0$ at the point $x = 1, y = 2$

ANSWER

1. $y - 3x - 2 = 0$

Make y subject of the formula

$y = 3x + 2$

Compare with $y = mx + c$

$m_1 = 3$

$3y + x + 9 = 0$

Make y subject of the formula

$3y = -x - 9 + 0$

Divide through by 3

$y = \frac{-x}{3} - \frac{9}{3}$

$y = \frac{-x}{3} - 3$

$y = -\frac{1}{3}x - 3$

Compare with $y = mx + c$

$m_2 = -\frac{1}{3}$

It is perpendicular when $m_1 \cdot m_2 = -1$

$m_1 = 3$ and $m_2 = \frac{-1}{3}$, $3 \cdot -\frac{1}{3} = -1$

Therefore it is perpendicular. → $y = 3x - 2 = 0$ and $3y + x + 9 = 0$ are perpendicular.

$\frac{dy}{dx} \Rightarrow 2x + 2y\frac{dy}{dx} + 3x\frac{dy}{dx} + 3y = 0$

Collect like terms

$2y\frac{dy}{dx} + 3x\frac{dy}{dx} = -2x - 3y$

$\left(\frac{dy}{dx}\right)\ 2y + 3x = -2x - 3y$

Divide through by $2y + 3x$

$\frac{dy}{dx} = \frac{-2x - 3y}{2y + 3x}$

$m = \frac{dy}{dx}$ | $x=1, y=2$ $\rightarrow \frac{-2(1) - 3(2)}{2(2) + 3(1)}$

$m = \frac{-2 - 6}{4 + 3} = \frac{-8}{7}$

$m = \frac{-8}{7}$

Equation of tangent

$y - y_1 = m(x - x_1)$

$y - 2 = \frac{-8}{7}(x - 1)$

$7(y - 2) = -8(x - 1)$

$7y - 14 = -8x + 8$

$7y + 8x - 14 - 8 = 0$

$7y + 8x - 22 = 0$

Equation of normal

$y - y_1 = \frac{-1}{m}(x - x_1)$

$y - 2 = \frac{-1}{-8/7}(x - 1)$

$y - 2 = \frac{7}{8}(x - 1)$

$$8(y-2) = 7(x-1)$$
$$8y - 16 = 7x - 7$$
$$8y - 7x - 16 + 7 = 0$$
$$8y - 7x - 9 = 0$$

to each other.

ii $3y - 4 = 2x + 3$

Make y subject of the formula

$3y = 2x + 3 + 4$

$3y = 2x + 7$

Divide through by 3

$y = \frac{2x}{3} + \frac{7}{3}$

Compare with $y = mx + c$

$m_1 = \frac{2}{3}$

$y - 5 = x + 6$

$y = x + 6 + 11$

$y = x + 11$

Compare with $y = mx + c$

$m_2 = 1$

Perpendicular; $m_1 \cdot m_2 = -1$

Parallel; $m_1 = m_2$

$m_1 \times m_2 = \frac{2}{3} \times 1 = \frac{2}{3}$

$\therefore m_1 m_2 \neq -1$

$m_1 = m_2 \;,\; \frac{2}{3} = 1$

$\therefore m_1 \neq m_2$

$3y - 4 = 2x + 3$ and $y - 5 = x + 6$ are ~~neither~~ not perpendicular ~~nor~~ ~~parallel~~.

3. $x^2 + y^2 + 3xy - 11 = 0 \qquad (1, 2)$

$m = \frac{dy}{dx}$

$x + 9 = 0$

$\frac{dy}{dx} = 2x + 2y\frac{dy}{dx} + 3\left(x \cdot \frac{dy}{dx} + y \cdot 1\right) = 0$