

1. $-y = \frac{1}{x-2}$
- The function is defined for all real numbers except $x = 2$
 - The domain is the set of real numbers except $x = 2$
 - The codomain of the set of real number except $y = 0$

2. $k = \ln v$
 $\frac{dk}{dv} = \frac{1}{v}$

3a) $2x - 3y - 2 = 0$
 $-3y = 2 - 2x$
 $y = \frac{2 - 2x}{-3}$
 $y = \frac{2x + 2}{3}; \frac{2}{3}(x+1)$

b) $x^2 + y^2 = 4$
 $y^2 = 4 - x^2$
 $y = \pm \sqrt{4 - x^2}$

4. Find $\frac{dp}{dt}$, $p = \sin^{-1} t$
 $p = \frac{t}{\sin}; t = \sin p$

$\frac{dt}{dp} = \cos p; \frac{dp}{dt} = \frac{1}{\cos p}$
 Recall, $\cos^2 y + \sin^2 y = 1$
 $\cos y = \sqrt{1 - \sin^2 y}$
 $t = \sin p$

$\therefore \cos p = \sqrt{1 - t^2}$
 Hence, $\frac{dp}{dt} = \frac{1}{\sqrt{1 - t^2}}$

5. $F(x) = 2x^2 - 5, g(x) = 4x - 2$
 $F \circ g(x) = 2(4x - 2)^2 - 5$
 $= 2(16x^2 - 16x + 4) - 5$
 $= 32x^2 - 32x + 8 - 5$
 $= 32x^2 - 32x + 3$

$g \circ f(x) = 4(2x^2 - 5) - 2$
 $= 8x^2 - 20 - 2$
 $= 8x^2 - 22$

6) Show that $f(x) = f_1(x) + f_2(x)$
 $f(x) = 3x^2 - 2x + 1$
 $f_1(x) = \frac{f(x) + f(-x)}{2}$

$f(-x) = 3(-x)^2 - 2(-x) + 1$
 $= 3x^2 + 2x + 1$

$f_1(x) = \frac{3x^2 - 2x + 1 + 3x^2 + 2x + 1}{2}$
 $= \frac{6x^2 + 2}{2} = 3x^2 + 1$

$f_2(x) = \frac{3x^2 - 2x + 1 - (3x^2 + 2x + 1)}{2}$
 $= \frac{-4x}{2} = -2x$

$f_1(x) + f_2(x) = 3x^2 + 1 - 2x$
 $= 3x^2 - 2x + 1$

7) Differentiate $y = \cos x$
 $y + \delta y = \cos(x + \delta x)$

$\delta y = \cos(x + \delta x) - \cos x$ ($y = \cos x$)

Recall
 $\cos(A+B) - \cos(A-B) = -2\sin A \sin B$ — (1)

Comparing (1) & (2)

$A+B = x + \delta x$ — (2)

$A-B = x$ — (3)

Adding (2) & (3) and subtracting (2) & (3)

$2A = 2x + \delta x \implies A = \frac{2x + \delta x}{2}$

$A = \frac{2x + \delta x}{2}$, Comparing (1) & (2)

$\delta y = \cos - \cos x$
 $= 2\sin\left(x + \frac{\delta x}{2}\right) \sin\left(\frac{\delta x}{2}\right)$

Dividing through by δx
 $\frac{\delta y}{\delta x} = \frac{2\sin\left(x + \frac{\delta x}{2}\right) \sin\left(\frac{\delta x}{2}\right)}{\delta x}$

$$\frac{dy}{dx} = \frac{-\sin(x + \delta x/a) \sin(\delta x/a)}{\delta x/a}$$

$$= -\sin(x + \delta x/a) \times \sin(\delta x/a)$$

Taking limit $\delta x \rightarrow 0$
 $\lim_{\delta x \rightarrow 0} \frac{\sin \delta x/a}{\delta x/a} = 1$

$$\frac{dy}{dx} = -\sin(x + 0/a) = -\sin x$$

$$\frac{dy}{dx} = -\sin x$$

8 $y = 3t^2$; $x = 1/t^2$
 $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$
 $= \frac{dy}{dt} \div \frac{dx}{dt}$

$$\frac{dy}{dt} = 6t \quad ; \quad \frac{dx}{dt} = \frac{-2}{t^3}$$

$$\frac{dy}{dx} = 6t \div \frac{-2}{t^3}$$

$$= 6t \times \frac{-t^3}{2} = \frac{-6t^4}{2}$$

$$= \frac{-12}{t^2}$$

9 $y = x^2 \cos 2x e^{4x}$
Soln

Taking Log of both sides
 $\ln y = \ln x^2 + \ln \cos 2x + \ln e^{4x}$

Differentiating both w.r.t x
 $\frac{1}{y} \frac{dy}{dx} = \frac{1}{x^2} (2x) + \frac{1}{\cos 2x} (-2 \sin 2x) + 4$
 $\frac{1}{y} \frac{dy}{dx} = \frac{2}{x} - \frac{2 \sin 2x}{\cos 2x} + 4$

Multiplying both sides by y
 $\frac{dy}{dx} = y \left(\frac{2}{x} - \frac{2 \sin 2x}{\cos 2x} + 4 \right)$
 $= x^2 \cos 2x e^{4x} \left(\frac{2}{x} - 2 \tan 2x + 4 \right)$

10 $y = \sin(3x^3 + 5)$

Let $u = 3x^3 + 5$

$$\frac{dy}{dx} = \cos x$$

du

$$\frac{du}{dx} = dx^2$$

dx

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= \cos u \times 9x^2$$

$$= 9x^2 \cos u$$

$$= 9x^2 \cos(3x^3 + 5)$$