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Computer science
19/sci02/087

Assignment

Find the integrals of the following

(a) $\int \frac{dx}{x^2+7}$

$\Rightarrow \int \frac{dx}{x^2+7} = \int \frac{dx}{7 \left(\frac{x^2}{7} + 1 \right)}$ ^{solution} $= \frac{1}{7} \int \frac{dx}{\frac{x^2}{7} + 1} = \frac{1}{7} \int \frac{dx}{\left(\frac{x}{\sqrt{7}} \right)^2 + 1}$

$$u = \frac{x}{\sqrt{7}}$$

(rationalize)

$$u = \frac{x}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{7}x}{7}$$

$$u = \frac{\sqrt{7}x}{7}$$

$$\frac{du}{dx} = \frac{\sqrt{7}}{7}$$

$$dx = \frac{7}{\sqrt{7}} du$$

$$= \frac{1}{7} \int \frac{1}{u^2+1} dx = \frac{1}{7} \int \frac{1}{u^2+1} \cdot \frac{7}{\sqrt{7}} du = \frac{7}{7\sqrt{7}} \int \frac{1}{u^2+1} du$$

Recall, $\int \frac{1}{u^2+1} du$ is a standard, so it equates to \arctan

$$= \frac{7}{7\sqrt{7}} \arctan(u) + C, \text{ Recall } u = \frac{\sqrt{7}x}{7}$$

$$= \frac{\sqrt{7}}{7} \arctan\left(\frac{\sqrt{7}x}{7}\right) + C$$

$$= \sqrt{7}x \arctan\left(\frac{\sqrt{7}x}{7}\right) + C$$

$$(b) \int \frac{dx}{x^2+64}$$

solution

$$\int \frac{dx}{x^2+64} = \int \frac{dx}{64 \left(\frac{x^2}{64} + 1 \right)} = \frac{1}{64} \int \frac{dx}{\frac{x^2}{64} + 1} = \frac{1}{64} \int \frac{du}{\left(\frac{x}{8} \right)^2 + 1}$$

$$u = \frac{x}{8}$$

$$\frac{du}{dx} = \frac{1}{8}$$

$$dx = 8 du$$

$$= \frac{1}{64} \int \frac{1}{u^2+1} dx = \frac{1}{64} \int \frac{1}{u^2+1} \cdot 8 du = \frac{1}{8} \int \frac{1}{u^2+1} du$$

Recall $\int \frac{1}{u^2+1} du$ is a standard integral, so it equates to $\arctan(u)$

$$\therefore = \frac{1}{8} \int \frac{1}{u^2+1} du = \frac{1}{8} \arctan(u), \text{ Recall } u = \frac{x}{8}$$

$$= \frac{\arctan\left(\frac{x}{8}\right)}{8} + C$$

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