

1. A particle moves along a curve $x = 7t^2$, $y = 6t^2 - 4t$, $z = t - 5$ where t is time. Find its velocity.

The position vector $r = xi + yj + zk$
 $r = (7t^2)i + (6t^2 - 4t)j + (t - 5)k$

$$\text{Velocity} = \frac{dr}{dt} = (14t)i + (12t - 4)j + k$$

2. If $A = i + 2j - 4k$, $B = 2i - 3j + k$, $C = 4j - 3k$ find $A \times (B \times C)$

$$(B \times C) = \begin{vmatrix} i & j & k \\ 2 & -3 & 1 \\ 0 & 4 & -3 \end{vmatrix}$$

$$(B \times C) = i \begin{vmatrix} -3 & 1 \\ 4 & -3 \end{vmatrix} - j \begin{vmatrix} 2 & 1 \\ 0 & -3 \end{vmatrix} + k \begin{vmatrix} 2 & -3 \\ 0 & 4 \end{vmatrix}$$

$$= i(9 - 4) - j(-6 - 0) + k(8 - 0)$$

$$= 5i + 6j + 8k$$

$$A \times (B \times C) = \begin{vmatrix} i & j & k \\ 1 & 2 & -4 \\ 5 & 6 & 8 \end{vmatrix}$$

$$A \times (B \times C) = i \begin{vmatrix} 2 & -4 \\ 6 & 8 \end{vmatrix} - j \begin{vmatrix} 1 & -4 \\ 5 & 6 \end{vmatrix} + k \begin{vmatrix} 1 & 2 \\ 5 & 6 \end{vmatrix}$$

$$= i(16 + 24) - j(6 + 20) + k(6 - 10)$$

$$= 40i - 26j - 4k$$

4. If $A = 7i + 2j - k$, $B = 2i + j + 4k$, $C = i + j + k$ find $(A + C) \cdot (B - A)$

$$(A + C) = 7i + i + 2j + j - k + k$$

$$= 8i + 3j$$

$$(B - A) = 2i - 7i + j - 2j + 4k - k = -5i - j + 3k$$

$$A+C = 8i + 3j$$

$$B-A = -5i - j + 3k$$

$$(A+C) \cdot (B-A) = (8i + 3j) \cdot (-5i - j + 3k)$$

$$= 40 - 3$$

$$= 37$$

5) Find a unit vector tangent to the space curve $x=t$, $y=t^2$, $z=t^3$ at point where $t=1$

$$r(t) = t i + t^2 j + t^3 k$$

Tangent vector

$$r'(t) = i + (2t)j + (3t^2)k$$

$$r'(1) = i + (2(1))j + (3(1)^2)k$$

$$r'(1) = i + 2j + 3k$$

$$\text{Unit tangent vector} = \frac{r'}{|r'|}$$

$$|r'| = \sqrt{(1)^2 + (2)^2 + (3)^2}$$

$$= \sqrt{1 + 4 + 9} = \sqrt{14}$$

$$\text{Unit tangent vector} = \frac{i + 2j + 3k}{\sqrt{14}}$$