19/ENG05/048 NDOKORIE PASCAL CHINAGOZI
MECHATRONICS ENGINEERING 02/04/2020

1) $\lim_{x \to 0} \left\{ \frac{x - \cos x}{x} \right\}$

Soln

$\lim_{x \to 0} \left\{ \cdot \frac{x - \cos x}{x} \right\}$

By direct substitution, we have $\frac{0}{0}$: Using L'hopital's rule.

$\lim_{x \to 0} \left\{ \frac{x - \cos x}{x} \right\} = \lim_{x \to 0} \left\{ \frac{1 - (-\sin x)}{1} \right\}$

$\therefore \lim_{x \to 0} \left\{ \frac{1 + \sin x}{1} \right\} = \frac{1 + \sin 0}{1} = \frac{1+0}{1} = 1$

2) $y = -3 \tan 7x e^{3x}$

Soln

$y = -3 \tan 7x e^{3x}$

$u = -3 \quad v = \tan 7x \quad w = e^{3x}$

$du/dx = 0 \quad dv/dx = 7\sec^2 7x ; \quad dw/dx = 3e^{3x}$

$\frac{dy}{dx} = y \left[\frac{1}{u} \times \frac{du}{dx} + \frac{1}{v} \times \frac{dv}{dx} + \frac{1}{w} \times \frac{dw}{dx} \right]$

$= y \left[\frac{1}{3} \times 0 + \frac{1}{\tan 7x} \times 7\sec^2 7x + \frac{1}{e^{3x}} \times 3e^{3x} \right]$

$= y \left[0 + \frac{7\sec^2 7x}{\tan 7x} + 3 \right]$

$dy/dx = -3 \tan 7x e^{3x} \left[\frac{7\sec^2 7x}{\tan 7x} + 3 \right]$

3) $y = \cos 3x$

Soln

$y = \cos 3x$

$y + \Delta y = \cos 3(x + \Delta x)$

$y + \Delta y = \cos 3x + \cos 3\Delta x$

$\Delta y = \cos(3x + 3\Delta x) - \cos 3x \quad -(i)$

Recall $\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2} \quad -(ii)$

19/BNG0510+S
MECHATRONICS ENGINEERING

NWOKORIE PASCAL CHINAMDI
02/0461202D

3 Cont'd)

Comparing eqn (i) & (ii)

$A = 3x + 3\Delta x \quad B = 3x$

$$\frac{A+B}{2} = \frac{3x + 3\Delta x + 3x}{2} = \frac{6x + 3\Delta x}{2} = 3x + \frac{3\Delta x}{2}$$

$$A - B = 3x + 3\Delta x - 3x = 3\Delta x$$

~~Hence $\Delta y = -2\sin(3x + 3\Delta x$~~

Hence $\Delta y = -2\sin\left(3x + \frac{3\Delta x}{2}\right)\sin\left(\frac{3\Delta x}{2}\right)$

Dividing through by Δx

$$\frac{\Delta y}{\Delta x} = \frac{-2\sin\left(3x + \frac{3\Delta x}{2}\right)\sin\left(\frac{3\Delta x}{2}\right)}{\Delta x}$$

Multiplying both numerator and denominator through by $3/2$

$$= \frac{-2\sin\left(3x + \frac{3\Delta x}{2}\right)\sin\left(\frac{3\Delta x}{2}\right) \times 3/2}{\Delta x \times 3/2}$$

$$\frac{\Delta y}{\Delta x} = \frac{-3\sin\left(3x + \frac{3\Delta x}{2}\right)\sin\left(\frac{3\Delta x}{2}\right)}{3\Delta x/2}$$

$\lim \Delta y/\Delta x = -3\sin 3x$

~~$\Delta \to$~~ $\Delta x \to 0$

$$\lim_{x \to 0} \frac{\sin 3\Delta x/2}{\frac{3\Delta x}{2}} = 1$$

$\therefore -3\sin 3x \times 1$

Hence $\left.\frac{dy}{dx}\right| = -3\sin 3x$

4) $f(x) = 2x^3 - 7x$

$g(x) = -3x$

$f(x) - g(x) = 2x^3 - 7x - (-3x)$

$= 2x^3 - 7x + 3x$

$= 2x^3 - 4x$

NWOKORIE PASCAL CHINAMDI
M/BME/05/043
MECHATRONICS ENGINEERING
02/04/2020

4 (cont'd)

$(f-g)(5) = (2x^3 - 4x)(5)$

$= 10x^3 - 20x$

dividing through by 10

$x^3 - 2x$

5) $f \circ g(x)$

$f(x) = 4x^2 + 2$

$g(x) = 2x + 3$

$f \circ g(x) = 4(2x+3)^2 + 2$

$= 4(4x^2 + 12x + 9) + 2$

$= 16x^2 + 48x + 36 + 2$

$f \circ g(x) = 16x^2 + 48x + 38$

~~$f \circ g(x) =$~~

6) $x^2 + 2xy + y^2 = 1020$

Soln

$x^2 + 2xy + y^2 = 1020$

$2x\frac{dx}{dx} + 2x\,dy/dx + 2y\frac{dx}{dx} + 2y\,dy/dx = 0$

$= 2x + 2x\,dy/dx + 2y + 2y\,dy/dx = 0$

$= 2x + 2y + dy/dx\,(2x + 2y) = 0$

$dy/dx\,(2x + 2y) = -2x - 2y$

$dy/dx = \frac{-2x - 2y}{2x + 2y}$

7) $y = x^2 \cos x$

19/ENG05/043 · NKOOKURIE PASCAL CHINEMANDU

MECHATRONICS ENGINEERING 02/04/2020

7 Cont'd)

Soln

Put $u = x^2$, $v = \cos 2x$

$du/dx = 2x$ $\quad dv/dx = -\sin 2x$

Recall $u \cdot du/dx + v \cdot dv/dx$

$= x^2 \times (-\sin x) + \cos x \times (2x)$

$= -x^2 \sin x + 2x \cos 2x$

$= \underline{2x \cos 2x}$

$= 2x \cos x - 2x^2 \sin x$