OLA-AMUDA IBUKUN MAYOWA
19/MHS11/105
PHARMACY
MAT 104

1) For what values of x is the function $y = \frac{1}{x-2}$ defined? State the domain and codomain.

Solution

Its not defined because its a fraction, because of the denominator.
The function is defined for all real numbers except $x = 2$
Domain = Real numbers except $x = 2$
Codomain = Real numbers except $y = 0$.

2) If $K = \ln v$; differentiate K.

$$\frac{dk}{dv} = \frac{1}{v}$$

3) Express y as an explicit function of x in the following.

a) $2x - 3y - 2 = 0$

b) $x^3 + y^2 = 4$

Soln

a) $2x - 3y - 2 = 0$

$2x - 3y = 2$

$-3y = 2 - 2x$

$y = \frac{2 - 2x}{-3}$ $\qquad y = \frac{-2 + 2x}{3}$

b) $x^2 + y^2 = 4$

$y^2 = 4 - x^2$

$y = \sqrt{4 - x^2}$

$y = \pm\sqrt{4 - x^2}$

4) If $P = \sin^{-1} t$, Find the derivative of P

$p = \frac{t}{\sin}$ $\qquad t = \sin p$ --- (1)

Recall that: $\sin^2 p + \cos^2 y = 1$ — (2)

$\frac{dt}{dp}$ of [I] = $\cos p$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x} - \frac{2\sin 2x}{\cos 2x} + 4$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x} - 2\tan 2x + 4$$

Multiplying both sides by y, we have

$$\frac{dy}{dx} = y\left(\frac{2}{x} - 2\tan 2x + 4\right)$$

but $y = x^2 \cos 2x e^{4x}$

$$\frac{dy}{dx} = x^2 \cos 2x e^{4x}\left(\frac{2}{x} - 2\tan 2x + 4\right)$$

10) $y = \sin(3x^3 + 5)$

Let $u = 3x^3 + 5$

$$y = \sin u$$

$$\frac{du}{dx} = 9x^2$$

$$\frac{dy}{du} = \cos u$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= \cos u \times 9x^2$$

$$= 9x^2 \cos u$$

but $u = 3x^3 + 5$

$$\frac{dy}{dx} = 9x^2 \cos(3x^3 + 5)$$

$$f_o = \frac{f(x) - f(-x)}{2}$$

$$= \frac{[3x^2 - 2x + 1] - [3x^2 + 2x + 1]}{2}$$

$$= \frac{3x^2 - 2x + 1 - 3x^2 - 2x - 1}{2}$$

$$= \frac{-4x}{2} = -2x$$

$f(x) = f_e(x) + f_o(x)$

$f(x) = 3x^2 + 1 - 2x$

$= 3x^2 - 2x + 1$

7) Differentiate $y = \cos x$ from first principle

Solution

$y = \cos x$

$y + \delta y = \cos(x + \delta x)$

Sub. y from both sides.

$\delta y = \cos[x + \delta x] - y$

$y = \cos x$

$\delta y = \cos[x + \delta x] - \cos x$ — (*)

Consider from trig.

$[\cos A + B] = \cos A \cos B - \sin A \sin B$

$[\cos A - B] = \cos A \cos B + \sin A \sin B$

$[\cos A + B] - [\cos A - B]$

$= -2 \sin A \sin B$ — (2*)

Compare equ * to 2*.

Let: $A + B = x + \delta x$ — (i)

$A - B = x$ — (ii)

Add equ (i) and (ii)

$2A = 2x + \delta x$

$$A = \frac{2x + \delta x}{2}$$

$A = x + \frac{\delta x}{2}$

$B = \frac{\delta x}{2}$ — 3*

Compare * and 2*

$\cos(x + \delta x) - \cos x = -2 \sin(x + \frac{\delta x}{2}) \sin[\frac{\delta x}{2}]$

$\delta y = -2 \sin[x + \frac{\delta x}{2}] \sin[\frac{\delta x}{2}]$

$$\frac{\delta y}{\delta x} = \frac{-2 \sin[x + \frac{\delta x}{2}] \sin[\frac{\delta x}{2}]}{\delta x}$$

$$\frac{\delta y}{\delta x} = -\frac{\sin(x + \frac{\delta x}{2}) \sin[\frac{\delta x}{2}]}{\frac{\delta x}{2}}$$

To find a standard limit.

$$\lim \frac{\sin[\frac{\delta x}{2}]}{\frac{\delta x}{2}} = 1$$

$$\lim = -\sin[x + \frac{\delta x}{2}] \cdot \frac{\sin \frac{\delta x}{2}}{\frac{\delta x}{2}}$$

$= \sin[x + 0] \cdot 1$

$= \sin x$

$\therefore \lim \frac{dy}{dx} = \frac{\delta y}{\delta x} = -\sin x$

$\therefore \frac{\delta y}{dx} = -\sin x$

8) Find $\frac{dy}{dx}$ if $y = 3t^2$ and $x = \frac{1}{t^2}$

If $y = 3t^2 \quad x = \frac{1}{t^2}$

$y = 3t^2, \ \frac{dy}{dt} = 6t$

$x = \frac{1}{t^2}, \ \frac{dx}{dt} = -2t^{-3} = \frac{-2}{t^3}$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{6t}{\frac{-2}{t^3}} = \frac{6t \times t^3}{-2}$$

$= 3t^4$

9) Find $\frac{dy}{dx}$, If $y = x^2 \cos 2x e^{4x}$

Solution

$\ln y = \ln(x^2 \cos 2x e^{4x})$

$\ln y = \ln x + \ln \cos 2x + \ln e^{4x}$

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}[\ln x^2] + \frac{d}{dx}[\ln \cos 2x] + \frac{d}{dx}(\ln e^{4x}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x^2}(2x) + \frac{1}{\cos 2x}[-2\sin 2x] + \frac{1}{e^{4x}}[4e^{4x}$$

From equ (2) $\sin^2 p + \cos^2 p = 1$

$\cos^2 p = 1 - \sin^2 p$

$\cos p = \sqrt{1 - \sin^2 p}$

$\cos p = \sqrt{1 - t^2}$

$\frac{dt}{dp} = \cos p = \sqrt{(1-t)^2}$

$\frac{dp}{dt} = \frac{1}{\sqrt{(1-t^2}}$

5) If $f(x) = 2x^2 - 5$ and $g(x] = 4x - 2$, find fog (x) and gof (x)

If $f(x) = 2x^2 - 5$

$g(x) = 4x - 2$

Soln

a) $(fog)x$

$f(g(x))$

$f(x) = 2x^2 - 5$

$g(x) = 4x - 2$

$(fog)x = f(g(x)$

$= f(4x-2]$

$2[4x - 2]^2 - 5$

$2[4x-2][4x-2] - 5$

$2[16x^2 + 4 - 8x - 8x] - 5$

$2[16x^2 - 16x + 4] - 5$

$32x^2 - 32x + 8 - 5$

$= 32x^2 - 32x + 3$ //

b) $(gof)x$

$g(f(x)]$

$f(x) = 2x^2 - 5$

$g(x] = 4x - 2$

$g(2x^2 - 5]$

$4[2x^2 - 5] - 2$

$8x^2 - 20 - 2$

$8x^2 - 22$ //

6) If $f(x) = 3x^2 - 2x + 1 = 0$. Show that $f_e(x] + f_o(x] = f(x)$

Solution

$f_e(x) = \frac{f(x) + f(-x)}{2}$

$\therefore f(-x] = 3(-x)^2 - 2(-x) + 1$

$= 3x^2 + 2x + 1$

$= \frac{3x^2 - 2x + 1 + 3x^2 + 2x + 1}{2}$

$= \frac{6x^2 + 2}{2} = 3x^2 + 1$