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DEPARTMENT: PHARMACY

MATRIC NO: 19/MHS11/049

COURSE CODE: MAT 104

SOLUTIONS TO ASSIGNMENT

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QUESTIONS AND ANSWERS /SOLUTIONS

1.) For what values of x is the function $y = 1/x-2$ defined? State the domain and co-domain?.

$y = \frac{1}{x-2}$

Answer : It is defined for all set of real numbers except two (2). Therefore, it is not defined for all set of real numbers because of its denominator.

Domain : All set of real numbers except 2 because $x = 0$

Co-domain : All set of real numbers except 2, because no value of x will give $y = 0$

2.) If $K = \ln V$, differentiate K

$K = \ln V$

recall $\frac{dK}{dv} = \frac{1}{V}$

$\therefore \frac{dk}{dv} = \frac{1}{V}$

3.) Express y as an explicit function of x in the following

a) $2x - 3y - 2 = 0$

$3y = 2x - 2$

$\therefore y = \frac{2x-2}{3}$

b.) $x^2 + y^2 = 4$

$y^2 = 4 - x^2$

$y = \sqrt{4 - x^2}$

$y = 2 - x$

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4.) If $P = \sin^{-1} t$, find the derivative of P

$P = \sin^{-1} t$

$P = \frac{t}{x \sin}$ $\quad t = \sin p$ —(*)

$\therefore \frac{dt}{dp} = \cos p$

but we a looking for dp/dt

$\therefore \frac{dp}{dt} = \frac{1}{\cos p}$

Recall identity

$\cos^2 p + \sin^2 p = 1$

$\cos p = \sqrt{1 - \sin^2 p}$ from equ (*) $t = \sin p$

$\cos p = \sqrt{1 - t^2}$

since $\cos p = \sqrt{1 - t^2}$ —(**)

$\therefore$ insert equ (**) into $\frac{dp}{dt} = \frac{1}{\cos p}$

$\frac{dp}{dt} = \frac{1}{\sqrt{1 - t^2}}$

$\therefore \frac{dp}{dt} = \frac{1}{\sqrt{1 - t^2}}$

5.) If $f(x) = 2x^2 - 5$, $g(x) = 4x - 2$, find $fog(x)$ & $gof(x)$

a.) $(fog)x = f(g(x))$

$f(g(x)) = f(4x-2)$

$f(4x-2) = 2((4x-2)(4x-2)) - 5$

$f(4x-2) = 2(16x^2 - 16x + 4) - 5$

$f(4x-2) = 32x^2 - 32x + 8 - 5$

$f(4x-2) = 32x^2 - 32x + 3$

b.) $gof(x) = g(f(x))$

$g(2x^2-5) = 4(2x^2-5) - 2$

$= 8x^2 - 20 - 2$

$g(2x^2-5) = 8x^2 - 22$

$g(2x^2-5) = 8x^2 - 22$

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6.) If $f(x) = 3x^2 - 2x + 1 = 0$, show that $f_e(x) + f_o(x) = f(x)$

Solution

find $f_e(x)$ where $f(x) = 3x^2 - 2x + 1$

$f_e(x) = \dfrac{f(x) + f(-x)}{2}$, since $f(-x)$ isn't given

$\therefore f(-x) = 3(-x)^2 - 2(-x) + 1 = 3x^2 + 2x + 1$

$f(-x) = 3x^2 + 2x + 1$

$f_e(x) = \dfrac{3x^2 - \cancel{2x} + 1 + 3x^2 + \cancel{2x} + 1}{2} = \dfrac{6x^2 + 2}{2}$

$= \dfrac{\cancel{2}(3x^2 + 1)}{\cancel{2}} = 3x^2 + 1$

$\therefore f_e(x) = 3x^2 + 1$

find $f_o(x)$

$f_o(x) = \dfrac{f(x) - f(-x)}{2} = \dfrac{3x^2 - 2x + 1 - (3x^2 + 2x + 1)}{2}$

$f_o(x) = \dfrac{\cancel{3x^2} - 2x \cancel{+1} \cancel{-3x^2} - 2x \cancel{-1}}{2} = -\dfrac{4x}{2} = -2x$

$\therefore f_o(x) = -2x$

To show that $f_e(x) + f_o(x) = f(x)$

$3x^2 + 1 + (-2x)$ rearrange

$3x^2 - 2x + 1 = f(x)$

It is correct ✓

7.) Differentiate $y = \cos x$ from 1st principle

$y = \cos x$

$y + \delta y = \cos(x + \delta x)$

$\delta y = \cos(x + \delta x) - y$

$\delta y = \cos(x + \delta x) - \cos x$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

$\cos(A+B) - \cos(A-B) = -2\sin A \sin B$

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$A + B = x + \delta x$ — (i)

$A - B = x$ — (ii)

$2A = 2x + \delta x$

$A = \dfrac{2x}{2} + \dfrac{\delta x}{2}$

$\boxed{A = x + \dfrac{\delta x}{2}}$

$A - B = x$

$B = A - x$

$B = \cancel{x} + \dfrac{\delta x}{2} - \cancel{x}$

$\boxed{B = \dfrac{\delta x}{2}}$

$\delta y = -2\sin A \sin B$

$\delta y = -2\sin\left(x + \dfrac{\delta x}{2}\right)\sin\left(\dfrac{\delta x}{2}\right)$

divide tru by δx

$\dfrac{\delta y}{\delta x} = \dfrac{-2\sin\left(x + \frac{\delta x}{2}\right)\sin\left(\frac{\delta x}{2}\right)}{\delta x}$

$\dfrac{\delta y}{\delta x} = \dfrac{-\sin\left(x + \frac{\delta x}{2}\right)\sin\left(\frac{\delta x}{2}\right)}{\frac{\delta x}{2}}$

$\dfrac{\delta y}{\delta x} = -\sin\left(x + \dfrac{\delta x}{2}\right)\dfrac{\sin\left(\frac{\delta x}{2}\right)}{\frac{\delta x}{2}}$

Limit $\dfrac{\sin \frac{\delta x}{2}}{\frac{\delta x}{2}} \to 1$

$\delta x \to 0$

$\dfrac{\delta y}{\delta x} = \dfrac{dy}{dx} = -\sin(x + 0) \cdot 1$

limit $\delta x \to 0$

$\dfrac{\delta y}{\delta x} = \dfrac{dy}{dx} = -\sin(x + 0) \cdot 1$

limit $\delta x \to 0$

$\dfrac{dy}{dx} = -\sin x$

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8) Find dy/dx 4 $y = 3t^2$ and $x = 1/t^2$

Solution

This is a parametric question $\therefore$ we make use of a parametric equation

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

If $y = 3t^2$

$$\frac{dy}{dt} = 2 \times 3t^{2-1}$$

$$\frac{dy}{dt} = 6t$$

$x = 1/t^2$

$$\frac{dx}{dt} = t^{-2}$$

$$dx/dt = -2t^{-2-1}$$

$$dx/dt = -2t^{-3}$$

$$\therefore \frac{dy}{dx} = \frac{6t}{-2t^{-3}} = -\frac{\cancel{6}t}{\cancel{2}t^{-3}} = \frac{-3t}{t^{-3}}$$

$$= -3t^{1+3}$$

$$\therefore \frac{dy}{dx} = -3t^4$$

9) Find dy/dx 4 $y = x^2 \cos 2x\, e^{4x}$

$$\ln y = \ln x^2 + \ln \cos 2x + \ln e^{4x}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}(x^2) + \frac{d}{dx}(\cos 2x) + \frac{d}{dx}(e^{4x})$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2\cancel{x}}{\cancel{x}^2} + \frac{-2\cancel{\cos 2x}}{\cancel{\cos 2x}} + \frac{4\cancel{e^{4x}}}{\cancel{e^{4x}}}$$

$$\frac{dy}{dx} = y\left[\frac{2}{x} - 2\cancel{x} + 4\right)$$

$$\frac{dy}{dx} = y\left[\frac{2}{x} + \frac{2}{1}\right] = y\left[\frac{2+2x}{x}\right]$$ recall $y = x^2 \cos 2x\, e^{4x}$

$$\frac{dy}{dx} = x^2 \cos 2x\, e^{4x}\left[\frac{2+2x}{x}\right]$$

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10) $y = \sin(3x^3 + 5)$ find the derivative of y

Solution

$y = \sin \underbrace{(3x^3+5)}_{u}$ Using Chain Rule

$u = 3x^3 + 5$

$du/dx = 9x^2$

$du/dx = 9x^2$

$y = \sin u$

$dy/du = \cos u$

Using Chain Rule

$$\therefore \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= \cancel{9x^2 \times \cos u}\ \cos u \times 9x^2$$

$$= \cos u \times 9x^2$$

$$= 9x^2 \cos u$$

recall $u = 3x^3+5$ $\therefore \frac{dy}{dx} = 9x^2 \cos(3x^3+5)$

$$\frac{dy}{dx} = 9x^2 \cos\left(3x^3+5\right)$$

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