

# Saved Photos

ASIBOLA TADFEER OLUWAFIDBA

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MAT 104

1) The function is defined for all real numbers except  $x=2$

Domain = Real numbers except  $x=2$

Codomain = Real " "  $y=0$

2) If  $K = \ln v$  differentiate  $K$

$$\frac{d}{dk} (\ln v) = \frac{1}{v}$$

3) a)  $2x - 3y - z = 0$

$$2x - z = 3y$$

$$y = \frac{2x - z}{3}$$

b)  $x^2 + y^2 = 4$

$$x^2 - 4 = -y^2$$

$$y = \pm \sqrt{x^2 - 4}$$

4) If  $P = \sin^{-1} t$  find the derivative of  $P$

$$P = \frac{t}{\sin}$$

$$t = \sin P \quad \text{--- (1)}$$

Recall that  $\sin^2 P + \cos^2 P = 1$  --- 2

$$\frac{dt}{dP} \text{ of (1)} = \cos P$$

$$\text{from (2) } \sin^2 p + \cos^2 p = 1$$

$$\cos^2 p = 1 - \sin^2 p$$

$$\cos p = \sqrt{1 - \sin^2 p}$$

$$\cos p = \sqrt{1 - t^2}$$

$$\frac{dt}{dp} = \cos p = \sqrt{1 - t^2}$$

$$\therefore \frac{dp}{dt} = \frac{1}{\sqrt{1 - t^2}}$$

$$\text{e) } F(x) = 2x^2 - 5 \quad g(x) = 4x - 2$$

$$\begin{aligned} \text{a) } F \circ g(x) &= F(4x - 2) = 2(4x - 2)^2 - 5 \\ &= 2(4x - 2)(4x + 2) - 5 \\ &= 2(16x^2 + 8x - 8x - 4) - 5 \\ &= 2(16x^2 - 4) - 5 \\ &= 32x^2 - 2 - 5 \\ &= 32x^2 - 13 \end{aligned}$$

$$\begin{aligned} \text{b) } g \circ F(x) &= g(2x^2 - 5) = 4(2x^2 - 5) - 2 \\ &= 8x^2 - 20 - 2 \\ &= 8x^2 - 22 \end{aligned}$$

$$\text{b) } F(x) = F_1(x) + F_2(x)$$

$$\text{if } F(x) = 3x^2 - 2x + 1$$

$$\rightarrow F_1(x) = \frac{F(x) + F(-x)}{2}$$

$$F(x) = 3x^2 - 2x + 1$$

$$F(-x) = 3(-x)^2 - 2(-x) + 1 = 3x^2 + 2x + 1$$

$$F_1(x) = \frac{(3x^2 - 2x + 1) + (3x^2 + 2x + 1)}{2}$$

$$= \frac{3x^2 - 2x + 1 + 3x^2 + 2x + 1}{2}$$

$$= \frac{6x^2 + 2}{2}$$

$$= \frac{6x^2 + 2}{2} = 3x^2 + 1$$

$$= 3x^2 + 1$$

$$\rightarrow F_2(x) = \frac{F(x) - F(-x)}{2}$$

$$F(x) = 3x^2 - 2x + 1$$

$$F(-x) = 3x^2 + 2x + 1$$

$$= \frac{(3x^2 - 2x + 1) - (3x^2 + 2x + 1)}{2}$$

$$= \frac{3x^2 - 2x + 1 - 3x^2 - 2x - 1}{2}$$

$$= \frac{-Ax}{2} = -2x$$

Recall:  $F(x) = F_1(x) + F_2(x)$

$$= f(x) - (3x^2 + 1) + (-2x)$$

$$= 3x^2 + 1 - 2x$$

$$= 3x^2 - 2x + 1$$

7.) Differentiate  $y = \cos x$

$$y + \Delta y = \cos(x + \Delta x)$$

$$\Delta y = \cos(x + \Delta x) - \cos x \quad (\text{y} = \cos x) \quad - (1)$$

Recall

$$\cos(A+B) - \cos(A-B) = 2 \sin A \sin B \quad - (2)$$

Compare (1) & (2)

$$A+B = x + \Delta x \quad - (3)$$

$$A-B = x \quad - (4)$$

Add (3) & (4) & Subtract (3) & (4)

$$2A = 2x + \Delta x \quad \& \quad B = \Delta x / 2$$

$$A = \frac{2x + \Delta x}{2}$$

$$A = x + \frac{\Delta x}{2}$$

Compare (1) & (2)

$$\Delta y = \cos(x + \Delta x) - \cos x$$

$$= 2 \sin(x + \Delta x / 2) \sin(\Delta x / 2)$$

Dividing through by  $\Delta x$

$$\frac{\Delta y}{\Delta x} = \frac{-2 \sin(x + \Delta x / 2) \sin(\Delta x / 2)}{\Delta x}$$

$$\frac{\Delta y}{\Delta x} = \frac{-\sin(x + \Delta x / 2) \sin(\Delta x / 2)}{\frac{\Delta x}{2}}$$

$$= -\sin(x + \Delta x / 2) \times \frac{\sin(\Delta x / 2)}{\Delta x / 2}$$

Taking Limit  $\Delta x \rightarrow 0$

$$\lim_{\Delta x \rightarrow 0} \frac{\sin(\Delta x / 2)}{\Delta x / 2} = 1$$

$$\Delta y / \Delta x = -\sin(x + \Delta x / 2) \times 1$$

$$\Delta x \rightarrow 0$$

$$\Delta y / \Delta x = -\sin x$$

$$8) y = 3t^2; x = \frac{1}{t}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$
$$= \frac{dy}{dt} \div \frac{dx}{dt}$$

$$\frac{dy}{dt} = 6t; \quad \frac{dx}{dt} = \frac{-2}{t^3}$$

$$\frac{dy}{dx} = 6t \div \frac{-2}{t^3}$$

$$= 6t \times \frac{-2}{t^3} = \frac{-12t}{t^3} = \frac{-12}{t^2}$$

$$9) y = x^2 \cos 2x + 4^x$$

Soln

Take logs of both sides

$$\ln y = \ln x^2 + \ln(\cos 2x) + \ln 4^x$$

Differentiating both with  $x$

$$\frac{1}{y} \frac{dy}{dx} = \frac{1}{x^2} (2x) + \frac{1}{\cos 2x} (-2 \sin 2x) + 4$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{2}{x} - \frac{2 \sin 2x}{\cos 2x} + 4$$

Multiply both sides by  $y$

$$\frac{dy}{dx} = y \left( \frac{2}{x} - \frac{2 \sin 2x}{\cos 2x} + 4 \right)$$

$$= x^2 \cos 2x \left( \frac{2}{x} - \frac{2 \sin 2x}{\cos 2x} + 4 \right)$$

$$10) y = \frac{1}{2} \sin(3x^2 + 5)$$

$$\text{Let } u = 3x^2 + 5$$

$$\frac{dy}{du} = \cos u$$

$$\frac{du}{dx} = 6x$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= \cos u \times 6x$$

$$= 6x \cos u$$

$$= 6x \cos(3x^2 + 5)$$