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MAT 104 Assignment

1. $y = \frac{1}{x-2}$

Function is defined at $x-2=0$

Hence $x=2$

Domain = all real numbers of x except 2

Co-domain = all real numbers of y

2. $K = \ln V$

Differentiation of K

$$\frac{d}{dK} (\ln V) = \frac{1}{V}$$

3a. $2x - 3y - 2 = 0$

$$2x - 3y = 2$$

Differentiate all factors

$$\frac{d}{dx} (2x) - \frac{d}{dx} (3y) = \frac{d}{dx} (2)$$

$$2 - 3 \frac{dy}{dx} = 0$$

$$2 = 3 \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2}{3}$$

b. $x^2 + y^2 = 4$

Differentiate all factors

$$\frac{d}{dx} (x^2) + \frac{d}{dx} (y^2) = \frac{d}{dx} (4)$$

$$2x + 2y \cdot \frac{dy}{dx} = 0$$

$$2y \frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y}$$

$$\frac{dy}{dx} = \frac{-x}{y}$$

$$4 \quad p = \sin^{-1} t$$

find dy/dx

$$p = \frac{t}{\sin} \quad : \quad t = \sin p$$

$$\frac{dt}{dp} = \cos p \quad : \quad \frac{dp}{dt} = \frac{1}{\cos p}$$

$$\text{Recall } \cos^2 p + \sin^2 p = 1$$

$$\cos^2 p = 1 - \sin^2 p$$

$$\cos p = \sqrt{1 - \sin^2 p}$$

$$\text{Since } t = \sin p$$

$$\cos p = \sqrt{1 - t}$$

$$\text{Hence } \frac{dp}{dt} = \frac{1}{\sqrt{1-t}}$$

$$5 \quad f(x) = 2x^2 - 5$$

$$g(x) = 4x - 2$$

$$a) \quad f \circ g(x)$$

$$\begin{aligned} f \circ g(x) &= 2(4x - 2)^2 - 5 \\ &= 2\{16x^2 - 16x + 4\} - 5 \\ &= 32x^2 - 32x + 8 - 5 \\ &= 32x^2 - 32x + 3 \end{aligned}$$

$$b) \quad g \circ f(x) = 4(2x^2 - 5) - 2$$

$$= 8x^2 - 20 - 2$$

$$= 8x^2 - 22$$

$$6 \quad f(x) = 3x^2 - 2x + 1 = 0$$

Show $f(x) = f_e(x) + f_o(x)$

$$f_o = \frac{f(x) + f(-x)}{2}$$

$$f(-x) = 3(-x)^2 - 2(-x) + 1$$

$$= 3x^2 + 2x + 1$$

$$f_e(x) = \frac{3x^2 - 2x + 1 + 3x^2 + 2x + 1}{2}$$

$$= \frac{3x^2 - 2x + 1 + 3x^2 + 2x + 1}{2}$$

$$= \frac{-4x}{2}$$

$$= -2x$$

Hence

$$f(x) = f_e(x) + f_o(x)$$

$$f(x) = 3x^2 + 1 + (-2x)$$

$$f(x) = 3x^2 - 2x + 1$$

7 $y = \cos x$

$$y + dy = \cos(x + dx)$$

Subtract y from both sides

$$dy = \cos(x + dx) - y$$

but $y = \cos x$

$$dy = \cos(x + dx) - \cos x \dots (*)$$

Consider from trig

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\cos(A+B) - \cos(A-B) = -2 \sin A \sin B \dots (**)$$

Compare (*) and (**)

Let

$$A+B = x+dx \dots (i)$$

$$A-B = x \dots (ii)$$

Adding (i) and (ii)

$$2A = 2x + dx$$

$$A = \frac{2x + dx}{2}$$

$$A = x + \frac{dx}{2}$$

$$B = \frac{dx}{2}$$

(***)

Compare Eqn (***) to Eqn (**)

$$\cos(x+dx) - \cos x = -2 \sin\left(x + \frac{dx}{2}\right) \sin\left(\frac{dx}{2}\right)$$

$$dy = -2 \sin\left(x + \frac{dy}{2}\right) \sin\left(\frac{dx}{2}\right)$$

Dividing through by dx

$$\frac{dy}{dx} = \frac{-2 \sin(x + dx/2) \sin(dx/2)}{dx}$$

$$\frac{dy}{dx} = \frac{-\sin(x + dx/2) \sin(dx/2)}{\frac{dx}{2}} \dots \text{ (***) }$$

Take lim of Eqn (***)

$$\lim_{dx \rightarrow 0} \frac{dy}{dx} = \frac{dy}{dx} = -\sin(x+0) \cdot 1$$

$$dx \rightarrow 0 \quad = -\sin x$$

$$\lim_{dx \rightarrow 0} \frac{dy}{dx} = \frac{dy}{dx} = -\sin x$$

8. $y = 3t^2$ $x = \frac{1}{2}t^2$ find dy/dx

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

$$\frac{dy}{dt} = 6t$$

$$\frac{dx}{dt} = -2t^{-3} = \frac{-2}{t^3}$$

$$\frac{dy}{dx} = 6t \div \frac{-2}{t^3} = 3t \times \frac{t^3}{-2}$$

$$\frac{dy}{dx} = -3t^4$$

9. $y = x^2 \cos 2x e^{4x}$

Take logarithm of both sides

$$\ln y = \ln x^2 + \ln \cos 2x + \ln e^{4x}$$

Differentiate both sides with x

$$\frac{1}{y} \frac{dy}{dx} = \frac{1}{x^2} (2x) + \frac{1}{\cos 2x} (-2 \sin 2x) + 4$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{2}{x} - \frac{2 \sin 2x}{\cos 2x} + 4$$

multiply through by y

$$\frac{dy}{dx} = y \left(\frac{2}{x} - \frac{2 \sin 2x}{\cos 2x} + 4 \right)$$

$$\sin \theta \cdot y = x^2 \cos 2x e^{4x}$$
$$\frac{dy}{dx} = x^2 \cos 2x e^{4x} \left(\frac{2}{x} - \frac{2 \sin 2x}{\cos 2x} + 4 \right)$$

10 $y = \sin(3x^3 + 5)$

$$u = 3x^3 + 5$$

$$y = \sin u$$

$$\frac{du}{dx} = 9x^2$$

$$\frac{dy}{du} = \cos u$$

$$\frac{dy}{dx} = \frac{du}{dx} \times \frac{dy}{du}$$

$$= 9x^2 \cos u$$

$$\frac{dy}{dx} = 9x^2 \cos u$$

$$\text{but } u = 3x^3 + 5$$

$$\frac{dy}{dx} = 9x^2 \cos 3x^3 + 5$$