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Course: MAT 104

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Assignment

1. $y = \frac{1}{x-2}$

where $\Rightarrow x - 2 = 0$

$x = 2$

$\therefore$ (i). The function is defined for all real numbers except 2
(ii). The domain is the set of all real numbers except 2
(iii). The co-domain is the set of all real numbers except $y=0$

2. $K = \ln V$

Using chain rule $\rightarrow \frac{dK}{dV} = \frac{dK}{du} \times \frac{du}{dV}$

Let $u = V \Rightarrow \frac{du}{dV} = 1$

$K = \ln u \Rightarrow \frac{dK}{du} = \frac{1}{u}$

$\frac{dK}{dV} = \frac{1}{u} \times 1 = \frac{1}{u}$

Recall: $u = V$

$\therefore \frac{dK}{dV} = \frac{1}{V}$

3(a). $2x - 3y - 2 = 0$

$3y = 2x - 2$

$y = \frac{2x-2}{3} \Rightarrow \frac{2}{3}(x-1)$

$\therefore y = \frac{2}{3}(x-1)$

(b). $x^2 + y^2 = 4$

$y^2 = 4 - x^2$

$y = \pm\sqrt{4-x^2}$

4. $P = \sin^{-1} t$, Find dP/dt

$P = \sin^{-1} t \Rightarrow \arcsin t$

$P = \frac{t}{\sin}$; $t = \sin p$ ...... (*)

differentiating both sides

$\frac{dt}{dp} = \cos P$

but we want dp/dt,

$\therefore dp/dt = \frac{1}{\cos P}$

Recall; $\cos^2 P + \sin^2 P = 1$

$\cos^2 P = 1 - \sin^2 P$

$\cos P = \sqrt{1 - \sin^2 P}$

but $\sin P = t \Rightarrow \sin^2 P = t^2$

$\therefore \cos P = \sqrt{1 - t^2}$

Hence,

$\frac{dP}{dt} = \frac{1}{\cos P} = \frac{1}{\sqrt{1-t^2}}$

5. $f(x) = 2x^2 - 5$

$g(x) = 4x - 2$

(i). $fog(x) = f(g(x))$

$f(x) = 2x^2 - 5$

$g(x) = 4x - 2$

$f(g(x)) = f(4x-2)$

$= 2(4x-2)^2 - 5$

$= 2(4x-2)(4x-2) - 5$

$= 2(16x^2 - 16x + 4) - 5$

$fog(x) = 32x^2 - 32x + 8 - 5$

$= 32x^2 - 32x + 3$

(ii). $gof(x) = g(f(x))$

$= g(2x^2 - 5)$

$= 4(2x^2 - 5) - 2$

$= 8x^2 - 20 - 2$

$gof(x) = 8x^2 - 22$

6. Show that $f(x) = f_e(x) + f_o(x)$

$f(x) = 3x^2 - 2x + 1$

$f_e(x) = \frac{f(x) + f(-x)}{2}$

$$f(x) = 3x^2 - 2x + 1$$
$$f(-x) = 3(-x)^2 - 2(-x) + 1$$
$$= 3x^2 + 2x + 1$$
$$f_e = \frac{3x^2 \cancel{- 2x} + 1 + 3x^2 \cancel{+ 2x} + 1}{2}$$
$$= \frac{6x^2 + 2}{2} \Rightarrow \frac{\cancel{2}(3x^2 + 1)}{\cancel{2}}$$
$$f_e(x) = 3x^2 + 1$$
$$f_o(x) = \frac{f(x) - f(x)}{2}$$
$$= \frac{3x^2 - 2x + 1 - (3x^2 + 2x + 1)}{2}$$
$$= \frac{\cancel{3x^2} - 2x \cancel{+1} - \cancel{3x^2} - 2x \cancel{-1}}{2}$$
$$f_o(x) = \frac{-\cancel{4}x}{\cancel{2}} = \underline{\underline{-2x}}$$
$$f(x) = f_e(x) + f_o(x)$$
$$= 3x^2 + 1 + (-2x)$$
$$= 3x^2 - 2x + 1$$

7. Differentiate $y = \cos x$, from first principle

$$y = \cos x$$
$$y + \delta y = \cos(x + \delta x)$$
subtract y from both sides
$$\delta y = \cos(x + \delta x) - y$$
but $y = \cos x$
$$\therefore \delta y = \cos(x + \delta x) - \cos x \text{ ------ } (*)$$
Recall from Trig.
$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$
$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$
subtracting from both sides of both equations
$$\cos(A+B) - \cos(A-B) = -2 \sin A \sin B \text{ ------ } (2*)$$
comparing both equations
let; $A + B = x + \delta x$ -- (i)
$A - B = x$ --- (ii)
adding (i) and (ii)

$(A+B)+(A-B) = x + \delta x + x$

$2A = 2x + \delta x$

$A = \frac{\cancel{2}x + \delta x}{\cancel{2}}$

$A = x + \delta x/2$ - - - - (3*)

subtracting (i) and (ii)

$(A+B)-(A-B) = \cancel{x} + \delta x - \cancel{x}$

$A+B-A+B = \delta x$

$2B = \delta x$

$B = \delta x/2$ - - - - (4*)

comparing * and 2*

$\cos(x+\delta x) - \cos x = -2\sin\left(x+\frac{\delta x}{2}\right)\sin\left(\frac{\delta x}{2}\right)$

$\delta y = -2\sin\left(x+\frac{\delta x}{2}\right)\sin\left(\frac{\delta x}{2}\right)$

dividing both sides with δx

$$\frac{\delta y}{\delta x} = \frac{-2\sin\left(x+\frac{\delta x}{2}\right)\sin\left(\frac{\delta x}{2}\right)}{\delta x}$$

$$\frac{\delta y}{\delta x} = \frac{-\sin\left(x+\frac{\delta x}{2}\right)\sin\left(\frac{\delta x}{2}\right)}{\delta x/2}$$

$$\frac{\delta y}{\delta x} = -\sin\left(x+\frac{\delta x}{2}\right)\frac{\sin\left(\frac{\delta x}{2}\right)}{\delta x/2} \quad \text{- - - - (5*)}$$

A standard limit

$$\lim_{\delta x \to 0} \frac{\sin\left(\frac{\delta x}{2}\right)}{\delta x/2} = 1$$

Finding limit of (5*) as $\delta x \to 0$

$$\lim_{\delta x \to 0}\frac{\delta y}{\delta x} = \lim_{\delta x \to 0} -\sin\left(x+\frac{\delta x}{2}\right)\frac{\sin\left(\frac{\delta x}{2}\right)}{\delta x/2}$$

$= -\sin(x+0)\cdot 1.$

$= -\sin x$

$$\lim_{\delta x \to 0}\frac{\delta y}{\delta x} = \frac{dy}{dx} = \underline{\underline{-\sin x}}$$

8. $y = 3t^2$; $x = \frac{1}{t^2}$

$y = 3t^2 \quad \therefore \frac{dy}{dt} = 6t$

$x = \frac{1}{t^2}$ $\therefore \frac{dx}{dt} = -2t^{-3}$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

$= \frac{6t}{-2t^{-3}}$ $\Rightarrow 6t \div -2t^{-3}$

$= 6t \times \frac{1}{-2t^{-3}}$ $\Rightarrow 6t \times -2t^{3}$

$\frac{dy}{dx} = -12t^4$

9. $y = x^2 \cos 2x \, e^{4x}$

Taking $\log_e$ of both sides

$\ln y = \ln x^2 + \ln \cos 2x + \ln e^{4x}$

differentiating both sides with x

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{\cancel{x^2}}(2\cancel{x}) + \frac{1}{\cos 2x}(-2\sin 2x) + \frac{1}{\cancel{e^{4x}}}(4\cancel{e^{4x}})$

$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x} - \frac{2\sin 2x}{\cos 2x} + 4$

$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x} - 2\tan 2x + 4$

multiplying both sides by y

$\frac{dy}{dx} = y\left(\frac{2}{x} - 2\tan 2x + 4\right)$

$\frac{dy}{dx} = x^2 \cos 2x \, e^{4x}\left(\frac{2}{x} - 2\tan 2x + 4\right)$

10. $y = \sin(3x^3 + 5)$

Using chain rule ; $dy/dx = dy/du \times du/dx$

Let $u = (3x^3 + 5)$ $\Rightarrow du/dx = 9x^2$

$\therefore y = \sin u$ $\Rightarrow dy/du = \cos u$

$dy/dx = \cos u \times 9x^2$; Recall $u = (3x^3+5)$

$= 9x^2 \cos u$

$\therefore \frac{dy}{dx} = 9x^2 \cos(3x^3+5)$