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pharmacy

Mat 104

1. $-y = \frac{1}{x-2}$

- The function is defined for all real numbers except $x = 2$
- The domain is the set of real numbers except $x = 2$
- The codomain is the set of real numbers except $y = 0$

2. $k = \ln v$

$$\frac{dk}{dv} = \frac{1}{v}$$

3. a) $2x - 3y - 2 = 0$

$$-3y = 2 - 2x$$

$$y = \frac{2-2x}{-3}$$

$$y = \frac{2x-2}{3};\ \frac{2}{3}(x-1)$$

b) $x^2 + y^2 = 4$

$$y^2 = 4 - x^2$$

$$y = \pm\sqrt{4-x^2}$$

4 find dp/dt, $p = \sin^{-1} t$

$p = \frac{t}{\sin}$; $t = \sin p$

$\frac{dt}{dp} = \cos p$; $\frac{dp}{dt} = 1/\cos p$

Recall, $\cos^2 y + \sin^2 y = 1$

$$\cos y = \pm\sqrt{1+\sin^2 y}$$

$$t = \sin p$$

$$\therefore \cos p = \sqrt{1-t^2}$$

$$\boxed{\text{Hence, } dp/dt = 1/(1-t^2)}$$

5 $f(x) = 2x^2 - 5$; $g(x) = 4x - 2$

$$f\circ g(x) = 2(4x-2)^2 - 5$$

$$= 2(16x^2 - 16x + 4) - 5$$

$$= 32x^2 - 32x + 8 - 5$$

$$= 32x^2 - 32x + 3$$

$$g\circ f(x) = 4(2x^2 - 5) - 2$$

$$= 8x^2 - 20 - 2$$

$$= 8x^2 - 22$$

6) Show that $f(x) = f_e(x) + f_o(x)$

$$f(x) = 3x^2 - 2x + 1$$

$$f_e(x) = \frac{f(x) + f(-x)}{2}$$

$$f(-x) = 3(-x)^2 - 2(-x) + 1$$

$$= 3x^2 + 2x + 1$$

$$f_e(x) = \frac{3x^2 - 2x + 1 + (3x^2 + 2x + 1)}{2}$$

$$= \frac{6x^2 + 2}{2} = 3x^2 + 1$$

$$f_o(x) = \frac{3x^2 - 2x + 1 - (3x^2 + 2x + 1)}{2}$$

$$= -4x/2 = -2x$$

$$f_e(x) + f_o(x) = 3x^2 + 1 - 2x$$

$$= 3x^2 - 2x + 1$$

7) Differentiate $y = \cos x$

$$y + dy = \cos(x + dx)$$

$$dy = \cos(x + dx) - \cos x \quad (y = \cos x) \quad -(1)$$

Recall,

$\cos(A+B) - \cos(A-B) = -2\sin A \sin B$ —(2)

Comparing (1) & (2)

$A + B = x + dx$ —(3)

$A - B = x$ —(4)

Adding (3) & (4) & subtracting (3) & (4)

$2A = 2x + dx$ & $\boxed{B = dx/2}$

$$A = \frac{2x + dx}{2}$$

$$\boxed{A = x + dx/2}$$

Comparing (1) & (2)

$$dy = \cos(dx) - \cos x$$

$$= 2\sin(x + dx/2)\sin(dx/2)$$

Dividing through by dx

$$\frac{dy}{dx} = \frac{-2\sin(x + dx/2)\sin(dx/2)}{dx}$$

$$\frac{dy}{dx} = \frac{-\sin(x + dx/2)\sin(dx/2)}{dx/2}$$

$$= -\sin(x + dx/2) \times \frac{\sin(dx/2)}{dx/2}$$

Taking limit $dx \to 0$

$$\lim_{dx \to 0} \frac{\sin dx/2}{dx/2} = 1$$

$$dy/dx = -\sin(x + 0/2) \times 1$$

$$\lim_{dx \to 0} dx$$

$$dy/dx = -\sin x$$

8 $y = 3t^2$; $x = 1/t^2$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= dy/dt \div dx/dt$$

$$\frac{dy}{dt} = 6t \;;\; \frac{dx}{dt} = \frac{-2}{t^3}$$

$$dy/dx = 6t \div -2/t^3$$

$$= 6\cancel{t} \times \frac{-2}{\cancel{t^3}} = \frac{-6 \times -2}{t^2} = \frac{12}{t^2}$$

9 $y = x^2 \cos 2x \, e^{4x}$

s s/n

Taking loge of both sides

$$\ln y = \ln x^2 + \ln \cos 2x + \ln e^{4x}$$

Differentiating both wrt x

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x^2}(2x) + \frac{1}{\cos 2x}(-2\sin 2x) \cancel{+4}$$

$$+ 4$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x} - \frac{2\sin 2x}{\cos 2x} + 4$$

Multiplying both sides by 'y'

$$\frac{dy}{dx} = y\left(\frac{2}{x} - \frac{2\sin 2x}{\cos x} + 4\right.$$

$$= x^2 \cos 2x \, e^{4x} \times \frac{2}{x} - \frac{2\sin x}{\cos x} + 4$$

10) $y = \sin(3x^3 + 5)$

Let $u = 3x^3 + 5$

$$dy/du = \cos u$$

$$du/dx = 9x^2$$

$$dy/dx = \frac{dy}{du} \times \frac{du}{dx}$$

$$= \cos u \times 9x^2$$

$$= 9x^2 \cos u$$

$$= 9x^2 \cos 3x^3 + 5$$