MADUBOM OLUCHI PRECIOUS

19/MHS06/020

MEDICAL LABORATORY SCIENCE

1) $y = \frac{1}{(x-2)}$

The function is defined for all real numbers except $x=2$

The domain is real no except $x=2$

The codomain is the set of real numbers except $y=0$

2) $k = \ln v \quad \frac{dk}{dv} = \frac{1}{v}$

3) a) $2x - 3y - 2 = 0$

$3y = 2x - 2$

$y = \frac{2x-2}{3}$ //

b) $x^2 + y^2 = 4$

$y^2 = 4 - x^2$

$y = \sqrt{4-x^2}$ //

4) $p = \sin^{-1} t$

4) $p = \sin^{-1} t$

$$p = \frac{t}{\sin}$$

$t = \sin p$ — (i)

differentiate both sides

$$\frac{dt}{dp} = \cos p$$

Recall $\sin^2 p + \cos^2 p = 1$

$$\cos^2 p = 1 - \sin^2 p$$

$$\cos p = \sqrt{1 - \sin^2 p}$$

in $\sin p = t$ (from equation (i))

$$\cos p = \sqrt{1 - t^2}$$

$$\frac{dt}{dp} = \cos p$$

$$\frac{dp}{dt} = \frac{1}{\cos p}$$

$$\frac{dp}{dt} = \frac{1}{\sqrt{1 - t^2}}$$

5) $f(x) = 2x^2 - 5 \quad g(x) = 4x - 2$

a) $fog(x) = f(g(x))$

$f(g(x)) = f(4x-2)$
$= 2(4x-2)^2 - 5$
$= 2(4x-2)(4x-2) - 5$

$fog(x) = 2(16x^2 - 16x + 4) - 5$
$= 32x^2 - 32x + 8 - 5$
$= 32x^2 - 32x + 3$ //

b) $gof(x) = g(f(x))$

$g(f(x)) = g(2x^2 - 5)$
$= 4(2x^2 - 5) - 2$
$= 8x^2 - 20 - 2$
$= 8x^2 - 22$ //

6) $f(x) = 3x^2 - 2x + 1 = 0$

$$f_e(x) = \frac{f(x) + f(-x)}{2}$$

$f(-x) = 3(-x)^2 - 2(-x) + 1$
$= 3x^2 + 2x + 1$

$f(x) = 3x^2 - 2x + 1$

$$f_e(x) = \frac{3x^2 - 2x + 1 + 3x^2 + 2x + 1}{2}$$

f $\quad f_e(x) = \frac{6x^2+2}{2} = \frac{2(3x^2+1)}{2}$

$= 3x^2+1$

$f_o(x) = \frac{f(x) - f(-x)}{2}$

$= \frac{3x^2-2x+1-(3x^2+2x+1)}{2}$

$= \frac{\cancel{3x^2}-2x+1-\cancel{3x^2}-2x-1}{2}$

$= \frac{-4x}{2} = -2x$

$f(x) = f_e(x) + f_o(x) =$

$= 3x^2+1+(-2x)$

$= 3x^2-2x+1$ //

7) $\quad y = \cos x$

$y + \delta y = \cos(x+\delta x)$

Subtract y from both sides

$y + \delta y - y = \cos(x+\delta x) - y$

but $y = \cos x$

$\delta y = \cos(x+\delta x) - \cos x$ — (1)

Consider from trig

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\cos(A+B) - \cos(A-B) = -2\sin A\sin B$ — (ii)

compare (i) and (ii)

Let

$A+B = x+\delta x$ — *

$A-B = x$ — 2*

$2A = 2x+\delta x$

$$\left.\begin{aligned} A &= \frac{2x+\delta x}{2} = x+\delta x/2 \\ B &= \delta x/2 \end{aligned}\right\} \text{(iii)}$$

Compare (i) and (ii)

$\cos(x+\delta x) - \cos x = -2\sin A\sin B$

$\delta y = -2\sin(x+\delta x/2)\sin(\delta x/2)$

$$\frac{\delta y}{\delta x} = \frac{-2\sin(x+\delta x/2)\sin(\delta x/2)}{\delta x}$$

$$\frac{\delta y}{\delta x} = \frac{-\sin(x+\delta x/2)\sin(\delta x/2)}{\delta x/2}$$

$$= -\sin(x+\delta x/2)\frac{\sin(\delta x/2)}{\delta x/2} \quad \text{— (iv)}$$

A standard limit

$$\lim \frac{\sin(\delta x/2)}{\delta x/2} = 1$$

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \lim_{\delta x \to 0} -\sin(x + \delta x/2) \frac{\sin(\delta x/2)}{\delta x/2}$$

$$= -\sin(x+0) \cdot 1$$

$$= -\sin x$$

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \frac{dy}{dx} = -\sin x$$

8) $dy/dx$ If $y = 3t^2$ $x = 1/t^2$

$y = 3t^2$ ; $dy/dx = 6t$

$x = \frac{1}{t^2}$ ; $\frac{dx}{dt} = -2t^{-3}$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{6t}{-2/t^3} = \frac{\cancel{6}^{3}t}{1} \times \frac{t^3}{-\cancel{2}_1}$$

$$\cancel{= 6t^4} \quad \cancel{= -3t^4}$$

$$= -3t^4$$

9) $y = x^2 \cos 2x\, e^{4x}$

find $\log_e$ of both sides

$\ln y = \ln(x^2 \cos 2x\, e^{4x})$

$\ln y = \ln x^2 + \ln \cos 2x + \ln e^{4x}$

differentiate both sides wrt $x$

$$\frac{d}{dx}(\ln y) = \frac{d}{dx}(\ln x^2) + \frac{d}{dx}(\ln \cos 2x) + \frac{d}{dx}(\ln e^{4x})$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x^2}(2x) + \frac{1}{\cos 2x}(-2\sin 2x) + \frac{1}{e^{4x}}(4e^{4x})$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x} \cancel{+} - \frac{2\sin 2x}{\cos 2x} + \frac{4e^{4x}}{e^{4x}}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x} - 2\tan 2x + 4$$

multiply both sides by $y$

$$\frac{dy}{dx} = y\left(\frac{2}{x} - 2\tan 2x + 4\right)$$

$$\frac{dy}{dx} = x^2 \cos 2x\, e^{4x}\left(\frac{2}{x} - 2\tan 2x + 4\right)$$

10) $y = \sin(3x^3 + 5)$

$u = 3x^3 + 5$ $\qquad$ $du/dx = 9x^2$

$y = \sin u$ $\qquad$ $dy/du = \cos u$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$= \cos u \times 9x^2$

$= 9x^2 \cos u$

$= 9x^2 \cos 3x^3 + 5$