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Pharmacy

Mat 104

$$\frac{dy}{dx} = \frac{-\sin(x+dx/2)\sin(dx/2)}{dx/2}$$

$$= -\sin(x+dx/2) \times \frac{\sin(dx/2)}{dx/2}$$

Taking limit $dx \to 0$

$$\lim_{dx \to 0} \frac{\sin dx/2}{dx/2} = 1$$

$$dy/dx = -\sin(x + 0/2) \times 1$$

$$\lim_{dx \to 0} dx$$

$$dy/dx = -\sin x$$

8 $y = 3t^2$; $x = 1/t^2$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$= dy/dt \div dx/dt$$

$$\frac{dy}{dt} = 6t \;;\; \frac{dx}{dt} = \frac{-2}{t^3}$$

$$dy/dx = 6t \div -2/t^3$$

$$= 6t \times \frac{-2}{t^3} = \frac{-6\times-2}{t^2} = \frac{12}{t^2}$$

9 $y = x^2\cos 2x\, e^{4x}$

$3\frac{1}{2}$

Taking loge of both sides

$\ln y = \ln x^2 + \ln\cos 2x + \ln e^{4x}$

Differentiating both wrt x

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x^2}(2x) + \frac{1}{\cos 2x}(-2\sin 2x) + 4$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x} - \frac{2\sin 2x}{\cos 2x} + 4$$

Multiplying both sides by 'y'

$$\frac{dy}{dx} = y\left(\frac{2}{x} - \frac{2\sin 2x}{\cos 2x} + 4\right)$$

$$= x^2\cos 2x\, e^{4x}\left(\frac{2}{x} - \frac{2\sin x}{\cos x} + 4\right)$$

10) $y = \sin(3x^3 + 5)$

Let $u = 3x^3 + 5$

$dy/du = \cos u$

$du/dx = 9x^2$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= \cos u \times 9x^2$$

$$= 9x^2\cos u$$

$$= 9x^2\cos 3x^3 + 5$$

1. $y = \frac{1}{x-2}$

- The function is defined for all real numbers except $x = 2$
- The domain is the set of real numbers except $x = 2$
- The codomain is the set of real numbers except $y = 0$

2. $k = \ln v$

$$\frac{dk}{dv} = \frac{1}{v}$$

3. a) $2x - 3y - 2 = 0$

$$-3y = 2 - 2x$$

$$y = \frac{2-2x}{-3}$$

$$y = \frac{2x-2}{3}; \frac{2}{3}(x-1)$$

b) $x^2 + y^2 = 4$

$$y^2 = 4 - x^2$$

$$y = \pm\sqrt{4-x^2}$$

4. find dp/dt, $p = \sin^{-1} t$

$p = \frac{t}{\sin}$; $t = \sin p - t$

$\frac{dt}{dp} = \cos p$; $\frac{dp}{dt} = {}^1/\cos p$

Recall, $\cos^2 y + \sin^2 y = 1$

$$\cos y = \pm\sqrt{1+\sin^2 y}$$

$t = \sin p$

$\therefore \cos p = \sqrt{1-t^2}$

$\boxed{\text{Hence, } dp/dt = {}^1/(1-t^2)}$

5. $f(x) = 2x^2 - 5$; $g(x) = 4x - 2$

$f \circ g(x) = 2(4x-2)^2 - 5$

$= 2(16x^2 - 16x + 4) - 5$

$= 32x^2 - 32x + 8 - 5$

$= 32x^2 - 32x + 3$

$g \circ f(x) = 4(2x^2-5) - 2$

$= 8x^2 - 20 - 2$

$= 8x^2 - 22$

6) Show that $f(x) = fe(x) + fo(x)$

$f(x) = 3x^2 - 2x + 1$

$$fe(x) = \frac{f(x) + f(-x)}{2}$$

$f(-x) = 3(-x)^2 - 2(-x) + 1$

$= 3x^2 + 2x + 1$

$$fe(x) = \frac{3x^2 - 2x + 1 + (3x^2 + 2x + 1)}{2}$$

$$= \frac{6x^2+2}{2} = 3x^2 + 1$$

$$fo(x) = \frac{3x^2 - 2x + 1 - (3x^2 + 2x + 1)}{2}$$

$$= -4x/2 = -2x$$

$fe(x) + fo(x) = 3x^2 + 1 - 2x$

$= 3x^2 - 2x + 1$

7) Differentiate $y = \cos x$

$y + dy = \cos(x + dx)$

$dy = \cos(x+dx) - \cos x \quad (y = \cos x)$ — (1)

Recall,

$\cos(A+B) - \cos(A-B) = -2\sin A \sin B$ — (2)

Comparing (1) & (2)

$A + B = x + dx$ — (3)

$A - B = x$ — (4)

Adding (3) & (4) & Subtracting (3) & (4)

$2A = 2x + dx$ & $\boxed{B = dx/2}$

$A = \frac{2x+dx}{2}$

$\boxed{A = x + dx/2}$

Comparing (1) & (2)

$dy = \cos(dx) - \cos x$

$= 2\sin(x + dx/2)\sin(dx/2)$

Dividing through by dx

$$\frac{dy}{dx} = \frac{-2\sin(x + dx/2)\sin(dx/2)}{dx}$$