3 Find the equations of the tangent and normal to the curve x^2 + y^2 + 3xy − 11 = 0 at the point x = 1, y = 2

$x^2 + y^2 + 3xy - 11 = 0$

$\frac{dy}{dx}$

$2x + 2y\frac{dy}{dx} + 3(x\cdot\frac{dy}{dx} + y\cdot 1) = 0$

$2x + 2y\frac{dy}{dx} + 3(x\cdot\frac{dy}{dx} + y) = 0$

$2x + 2y\frac{dy}{dx} + 3x\frac{dy}{dx} + 3y = 0$

$2y\frac{dy}{dx} + 3x\frac{dy}{dx} = -2x - 3y \quad \frac{dy}{dx}(2y + 3x) = -2x - 3y$

$\frac{dy}{dx} = \frac{-2x - 3y}{2y + 3x}$

$m = \frac{dy}{dx}\Big|_{x=1, y=2} = \frac{-2(1) - 3(2)}{2(2) + 3(1)} = \frac{-2-6}{4+3} = \frac{-8}{7}$

3a Equation of the tangent to a curve

$y - y_1 = m(x - x_1)$

$y - 2 = -\frac{8}{7}(x - 1)$

$y - 2 = \frac{-8x}{7} + \frac{8}{7}$

$7y - 14 = -8x + 8$

$8x + 7y - 14 - 8 = 0$

$8x + 7y - 22 = 0$

3b Equation of the normal to a curve

$y - y_1 = -1/m(x - x_1)$

$y - 2 = -1/\frac{8}{7}(x - 1)$

$y - 2 = 7/8(x - 1)$

$y - 2 = \frac{7x}{8} - \frac{7}{8}$

$8y - 16 = 7x - 7$

$8y = 7x - 7 + 16$

~~$7x - 8y + 9 = 0$~~

$8y - 7x - 9 = 0$ // which gives the equation of the normal

Adeniyan Damola
19/Mhs01/034
S/n: 141
Medicine & Surgery

1. $y - 3x - 2 = 0$ and $3y + x + 9 = 0$

Solution

$y - 3x - 2 = 0$ and $3y + x + 9 = 0$

let $A = y - 3x - 2 = 0$

$\frac{dy}{dx} - 3 - 0 = 0$

$\frac{dy}{dx} - 3 = 0$

$m_1 = \frac{dy}{dx} = 3$

let $B = 3y + x + 9 = 0$

$3\frac{dy}{dx} + 1 + 0 = 0$

$3\frac{dy}{dx} + 1 = 0$

$m_2 = \frac{-1}{3}$

The product $= m_1 - m_2$

$3 \times \frac{-1}{3} = -1$

$y - 3x - 2 = 0$ and $3y + x + 9$ are perpendicular

2 $3y - 4 = 2x + 3$ and $y - 5 = x + 6$

let $A = 3y - 4 = 2x + 3$

$3\frac{dy}{dx} - 0 = 2 + 0$

$3\frac{dy}{dx} = 2$

$m_1 = \frac{dy}{dx} = \frac{2}{3}$

let $b = y - 5 = x + 6$

$\frac{dy}{dx} - 0 = 1 + 0$

$\frac{dy}{dx} = 0 + 1 = 1$

$m_2 = A \neq B$

i.e product of two slope

$\frac{2}{3} \times 1 = \frac{2}{3}$

Two lines are not perpendicular