MAT 102

If $A = 2i - j$, $B = 3i + j - 11k$ and $C = 4i + 4j - 5k$ find the following

i) $-3A + 7B - 8C$

ii) If $k = 2A + 4B - C$, find the direction cosine of k

iii) $A \times (B \times C)$

iv) $(2A \times B) \cdot (A \times 2B)$

v) $A \quad 2B - C$

Solution

i) $-3A + 7B - 8C$

$-3(2i - j) + 7(3i + j - 11k) - 8(4i + 4j - 5k)$

$(-6i + 3j) + (21i + 7j - 77k) - 32i - 32j + 40k$

$-6i + 21i - 32i + 3j + 7j - 32j - 77k + 40k$

$-17i - 22j - 37k$

ii) $k = 2A + 4B - C$

$k = 2(2i - j) + 4(3i + j - 11k) - (4i + 4j - 5k)$

$k = (4i - 2j) + (12i + 4j - 44k) - (4i + 4j - 5k)$

$k = (4i + 12i - 4i) - 2j + 4j + 4j - 44k - 5k$

$k = 12i + 6j - 49k$

$|k| = \sqrt{12^2 + (6)^2 - (49)^2}$

$|k| = \sqrt{144 + 36 + 2401}$

$|k| = \sqrt{2581}$

$|k| = 50.804$

direction Cosine of the above

$l = \cos\alpha = \dfrac{12}{50.804}$

$m = \cos\beta = \dfrac{6}{50.804}$

$n = \cos\gamma = \dfrac{49}{50.804}$

iii) $A \times (B \times C)$

$$B \times C = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 1 & -11 \\ 4 & 4 & -5 \end{vmatrix}$$

$$= i \begin{vmatrix} 1 & -11 \\ 4 & -5 \end{vmatrix} - j \begin{vmatrix} 3 & -11 \\ 4 & -5 \end{vmatrix} + k \begin{vmatrix} 3 & 1 \\ 4 & 4 \end{vmatrix}$$

$i[(1 \times -5) - (-11 \times 4)] - j[(3 \times -5) - (-11 \times 4)] + k[(3 \times 4) - (1 \times 4)]$

$i[-5 + 44] - j[-15 + 44] + k[12 - 4]$

$39i - 29j + 8k$

$$A \times (B \times C) \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & 0 \\ 39 & -29 & 8 \end{vmatrix}$$

$$= i \begin{vmatrix} -1 & 0 \\ -29 & 8 \end{vmatrix} - j \begin{vmatrix} 2 & 0 \\ 39 & 8 \end{vmatrix} + k \begin{vmatrix} 2 & -1 \\ 39 & -29 \end{vmatrix}$$

$i[(-1 \times 8) - (0 \times -29)] - j[(2 \times 8) - (0 \times 39)] + k[(2 \times -29) - (-1 \times 39)]$

$i[8 - 0] - j[16 - 0] + k[-58 + 39]$

$8i - 16j - 19k$

iv) $(3A \times B) \cdot (A \times 2B)$

$3(2i - j) = 6i - 3j$; $B = 3i + j - 11k$

$$(3A \times B) \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 6 & -3 & 0 \\ 3 & 1 & -11 \end{vmatrix}$$

$$i \begin{vmatrix} -3 & 0 \\ 1 & -11 \end{vmatrix} - j \begin{vmatrix} 6 & 0 \\ 3 & -11 \end{vmatrix} + k \begin{vmatrix} 6 & -3 \\ 3 & 1 \end{vmatrix}$$

$i[(-3 \times -11) - (0 \times 1)] - j[(6 \times -11) - (0 \times 3)] + k[(6 \times 1) - (-3 \times 3)]$

$i[33 - 0] - j[-66 - 0] + k[6 + 9]$

$33i + 66j + 15k$

Continuation of IV

$(A \times 2B)$

$A = 2i - j$; $2(3i + j - 11k) = 6i + 2j - 22k$

$(A \times 2B)$ $\begin{vmatrix} i & j & k \\ 2 & -1 & 0 \\ 6 & 2 & -22 \end{vmatrix}$

$i[(-1 \times 2) - (2 \times 22)] + j[(2 \times 2) - (6 \times -22)] + k[(2 \times -1) - (6 \times 2)]$

$i[0 -$

$i \begin{vmatrix} -1 & 0 \\ 2 & -22 \end{vmatrix} - j \begin{vmatrix} 2 & 0 \\ 6 & -22 \end{vmatrix} + k \begin{vmatrix} 2 & -1 \\ 6 & 2 \end{vmatrix}$

$i[(-1 \times -22) - (0 \times 2)] - j[(2 \times -22) - (0 \times 6)] + k[(2 \times 2) - (-1 \times 6)]$

$i[22 - 0] - j[-44 - 0] + k[4 + 6]$

$22i + 44j + 10k$

$(3A \times B) \cdot (A \times 2B)$ $\begin{vmatrix} i & j & k \\ 33 & 66 & 15 \\ 22 & 44 & 10 \end{vmatrix}$

$33 \begin{vmatrix} 44 & 10 \end{vmatrix} - 66 \begin{vmatrix} 22 & 10 \end{vmatrix} + 15 \begin{vmatrix} 22 & 44 \end{vmatrix}$

~~33 [44 × 10]~~

$33[44 \times 10] - 66[22 \times 10] + 15[22 \times 44]$

$33[440] - 66[220] + 15[968]$

$14520 - 14520 + 14520 = 14520$

v) $A = 2B - C$

$A = 2i - j$;

$2B = 2(3i + j - 11k) = 6i + 2j - 22k$

$C = 4i + 4j - 5k$

$(2i - j) - (6i + 2j - 22k) - (4i + 4j - 5k)$

$(2i - 6i - 4i) + (j + 2j + 4j) + (-22k - 5k)$

$-8i + 7j - 27k$

2) Definition of perpendicular and Co-planar Vectors.

Perpendicular is defined as a line meeting another at a right angle or 90°.

· Co-planar Vectors is defined as a Vectors parallel to the same plane, or lie on the same plane.