

JIBRI SALIM YUSUF  
19/MHS11/076  
PHARMACY  
MAT 104

1.  $y = \frac{1}{2x-2}$
- The function is defined for all real numbers except  $x=2$
  - The domain is the set of real numbers except  $x=2$
  - The Co-domain is the set of real numbers except  $y=0$ .

2.  $y = \ln v$   
 $\frac{dy}{dv} = \frac{1}{v}$

3. a)  $2x - 3y - 2 = 0$   
 $-3y = 2 - 2x$   
 $y = \frac{2 - 2x}{-3}$

$$y = \frac{2x - 2}{3}; \frac{2}{3}(x - 1)$$

b)  $x^2 + y^2 = 4$

$$y^2 = 4 - x^2$$

$$y = \pm \sqrt{4 - x^2}$$

4) find  $\frac{dp}{dt}$ ;  $p = \sin^{-1} t$   
 $p = \frac{t}{\sin}$ ;  $t = \sin p$  — 0  
 $\frac{dt}{dp} = \cos p$ ;  $\frac{dp}{dt} = \frac{1}{\cos p}$

Recall,  $\cos^2 y + \sin^2 y = 1$

$$\cos y = \pm \sqrt{1 - \sin^2 y} \quad t = \sin p$$

$$\therefore \cos p = \sqrt{1 - t^2}$$

Hence,  $\frac{dp}{dt} = \frac{1}{\sqrt{1 - t^2}}$

5.  $f(x) = 2x^2 - 5$ ;  $g(x) = 4x - 2$

$$f \circ g(x) = 2(4x - 2)^2 - 5$$

$$= 2(16x^2 - 16x + 4) - 5$$

$$= 32x^2 - 32x + 8 - 5$$

$$= 32x^2 - 32x + 3$$

$$f \circ f(x) = 4(2x^2 - 5)^2 - 2$$

$$= 8x^2 - 20 - 2$$

$$= 8x^2 - 22$$

6) Show that  $f'(0) = f'_e(x) + f'_o(x)$

$$f(x) = 3x^2 - 2x + 1$$

$$f_e(x) = \frac{f(x) + f(-x)}{2}$$

$$f(-x) = 3(-x)^2 - 2(-x) + 1$$

$$3x^2 + 2x + 1$$

$$f_e(x) = \frac{3x^2 - 2x + 1 + (3x^2 + 2x + 1)}{2}$$

$$= \frac{6x^2 + 2}{2} = 3x^2 + 1$$

$$f'(0) = \frac{3x^2 - 2x + 1 - (3x^2 + 2x + 1)}{2}$$

$$= \frac{-4x}{2} = -2x$$

$$f'_e(x) + f'_o(x) = 3x^2 + 1 - 2x$$

$$= 3x^2 - 2x + 1$$

7) Differentiate  $y = \cos x$

$$y + dy = \cos(x + \delta x)$$

$$\delta y = \cos(x + \delta x) - \cos x \quad (y = \cos x) \quad \text{--- (1)}$$

Recall,

$$\cos(A+B) - \cos(A-B) = -2\sin A \sin B \quad \text{--- (2)}$$

Comparing (1) & (2)

$$A+B = x + \delta x \quad \text{--- (3)}$$

$$A-B = x \quad \text{--- (4)}$$

Adding (3) & (4) & subtracting (3) & (4)

$$2A = 2x + \delta x$$

$$A = \frac{2x + \delta x}{2} \quad \& \quad B = \frac{\delta x}{2}$$

$$A = x + \frac{\delta x}{2}$$

Comparing (1) & (2)

$$\delta y = \cos(x + \delta x) - \cos x$$

$$= 2\sin(x + \frac{\delta x}{2}) \sin(\frac{\delta x}{2})$$

Dividing through by  $\delta x$

$$\frac{\delta y}{\delta x} = \frac{2\sin(x + \frac{\delta x}{2}) \sin(\frac{\delta x}{2})}{\delta x}$$

$\delta x$

$$\frac{dy}{dx} = \frac{\sin(x + \frac{\delta x}{2}) \sin(\frac{\delta x}{2})}{\delta x/2}$$

$$= \sin(x + \frac{\delta x}{2}) \times \frac{\sin(\frac{\delta x}{2})}{\delta x/2}$$

Taking limit  $\delta x \rightarrow 0$

$$\lim_{\delta x \rightarrow 0} \frac{\sin \delta x/2}{\delta x/2} = 1$$

$$\lim_{\delta x \rightarrow 0} \frac{dy}{dx} = -\sin(x + 0/2) \times 1$$

$$\frac{dy}{dx} = -\sin x$$

8.  $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$$= \frac{dy}{dt} \div \frac{dx}{dt}$$

$\frac{dy}{dt} = 6t$  ;  $\frac{dx}{dt} = \frac{-2}{t^3}$

$$\frac{dy}{dx} = 6t \div \frac{-2}{t^3}$$

$$= 6t \times \frac{-2}{t^3} = \frac{6t \cdot -2}{t^3} = \frac{-12}{t^2}$$

$$\frac{dy}{dx} = \frac{-12}{t^2}$$

9.  $y = x^2 \cos 2x \cdot e^{4x}$

Soln

Taking log of both sides

$$\ln y = \ln x^2 + \ln \cos 2x + \ln e^{4x}$$

Differentiating both with respect to  $x$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{1}{x^2} (2x) + \frac{1}{\cos 2x} (-2 \sin 2x) + 4$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{2}{x} - \frac{2 \sin 2x}{\cos 2x} + 4$$

Multiplying both sides by  $y$

$$\frac{dy}{dx} = y \left( \frac{2}{x} - \frac{2 \sin 2x}{\cos 2x} + 4 \right)$$

$$= x^2 \cos 2x e^{4x} \times \left( \frac{2}{x} - \frac{2 \sin 2x}{\cos 2x} + 4 \right)$$

10.  $y = \sin(3x^3 + 5)$

Let  $u = 3x^3 + 5$

$$\frac{dy}{du} = \cos u$$

$$\frac{dy}{dx} = 9x^2$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= \cos u \times 9x^2$$

$$= 9x^2 \cos u$$

$$= 9x^2 \cos 3x^3 + 5$$