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19/MHS11/034 - PHARMACY-MHS
MAT 104

1) for $y = \frac{1}{(x-2)}$

- the function is defined for all real numbers except $x=2$
- The domain is the set of real numbers except $x=2$ i.e $\mathbb{R} \setminus \{2\}$
- The codomain is the set of real numbers except $x=0$ i.e $\mathbb{R} \setminus \{0\}$

2) $K = \ln V$

$e^K = V$

$V = e^K$

$\frac{dv}{dk} = e^K$

$\therefore \frac{dk}{dv} = (e^K)^{-1}$

Recall $e^K = V$

$\therefore \frac{dK}{dv} = V^{-1} = \frac{1}{V}$

3) a) $2x - 3y - 2 = 0$

$-3y = -2x + 2$

$y = \frac{-2x+2}{-3}$

$\frac{-2(x-1)}{-3} = \frac{2(x-1)}{3}$

b) $x^2 + y^2 = 4$

$y^2 = 4 - x^2$

$\therefore y = \pm\sqrt{4-x^2}$

4) $P = \sin^{-1} t$

$\therefore t = \sin P$

$\frac{dt}{dp} = \cos P$

$\therefore \frac{dp}{dt} = \frac{1}{\cos P}$

Recall

$\cos^2 P + \sin^2 P = 1$

$\therefore \cos P = \sqrt{1 - \sin^2 P}$

But

$t = \sin P$

$\therefore \cos P = \sqrt{1 - t^2}$

$\therefore \frac{dp}{dt} = \frac{1}{\cos P} = \frac{1}{\sqrt{1-t^2}}$

5) $f(x) = 2x^2 - 5$

$g(x) = 4x - 2$

i) $f \circ g(x) = f(g(x))$

$= f(4x-2)$

$= 2(4x-2)^2 - 5$

$= 2(16x^2 - 16x + 4) - 5$

$32x^2 - 32x + 8 - 5$

$= 32x^2 - 32x + 3$

ii $g \circ f(x) = g(f(x))$

$= g(2x^2 - 5)$

$= 4(2x^2-5) - 2$

$= 8x^2 - 20 - 2$

$= 8x^2 - 22$

5) $f(x) = 3x^2 - 2x + 1$

$f(-x) = 3(-x)^2 - 2(-x) + 1$

$f(-x) = 3x^2 + 2x + 1$

$\therefore$ given

$f_e(x) = \dfrac{f(x) + f(-x)}{2}$

$= \dfrac{3x^2 - \cancel{2x} + 1 + 3x^2 + \cancel{2x} + 1}{2}$

$= \dfrac{6x^2 + 2}{2} = \dfrac{\cancel{2}(3x^2+1)}{\cancel{2}}$

$f_e(x) = 3x^2 + 1$

Given

$f_o(x) = \dfrac{f(x) - f(-x)}{2}$

$= \dfrac{3x^2 - 2x + 1 - (3x^2 + 2x + 1)}{2}$

$= \dfrac{\cancel{3x^2} - 2x + \cancel{1} - \cancel{3x^2} - 2x - \cancel{1}}{2}$

$= \dfrac{-2x - 2x}{2} = \dfrac{-4x}{2}$

$= -2x$

$\therefore f_e + f_o = (3x^2 + 1) - 2x$

$= 3x^2 - 2x + 1$

$\therefore f_x = f_e(x) + f_o(x)$

7) $y = \cos x$

$y + \delta y = \cos(x + \delta x)$

Subtract y from both sides

$\delta y = \cos(x + \delta x) - y$

But $y = \cos x$

$\delta y = \cos(x + \delta x) - \cos x$ --- (1)

Consider from trig

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$\cos(A-B) = \cos A \cos B + \sin A \sin B$

$\therefore \cos(A+B) - \cos(A-B) = -2\sin A \sin B$ --- (2)

Compare equ(1) and equ(2)

Let

$x + \delta x = A + B$ --- (i)

$x = A - B$ --- (ii)

Subtract equ (ii) from (i)

$2B = \delta x$

$\therefore \boxed{B = \dfrac{\delta x}{2}}$

$\boxed{A = \dfrac{\delta x}{2} + x}$

Comparing (1) and (2)

$\cos(x + \delta x) - \cos(x)$

$= -2\sin\left(\frac{\delta x}{2} + x\right)\sin\left(\frac{\delta x}{2}\right)$

$\therefore \delta y = -2\sin\left(\frac{\delta x}{2} + x\right)\sin\left(\frac{\delta x}{2}\right)$

Dividing by δx and 2

$\dfrac{\delta y}{\delta x} = \dfrac{-\cancel{2}\sin\left(\frac{\delta x}{2} + x\right)\sin\left(\frac{\delta x}{2}\right)}{\frac{\delta x}{2}}$

Remember

$$\lim_{\delta x \to 0} \frac{\sin\left(\frac{\delta x}{2}\right)}{\frac{\delta x}{2}} = 1$$

$$\therefore \frac{dy}{dx} = \lim_{\delta x \to 0} -\sin\left(\frac{\delta x}{2} + x\right) \cdot \frac{\sin\left(\frac{\delta x}{2}\right)}{\frac{\delta x}{2}}$$

$$= -\sin(0 + x) \cdot 1$$

$$\therefore \frac{dy}{dx} = -\sin x$$

8.) for $y = 3t^2$

$$\frac{dy}{dt} = 6t$$

for $x = 1/t^2 = t^{-2}$

$$\frac{dx}{dt} = -2t^{-3}$$

$$\therefore \frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dx} = \frac{6t}{-2t^{-3}}$$

$$= -3t^4$$

9.) For

$$y = x^2 \cos 2x \, e^{4x}$$

$$\ln y = \ln x^2 + \ln \cos 2x + 4x$$

$$\frac{1}{y}\left(\frac{dy}{dx}\right) = \frac{1}{x^2}(2x) + \frac{-2\sin 2x}{\cos 2x} + 4$$

$$\frac{1}{y}\left(\frac{dy}{dx}\right) = \frac{2}{x} + \frac{-2\sin 2x}{\cos 2x} + 4$$

Multiply through by y

$$\frac{dy}{dx} = y\left(\frac{2}{x} - \frac{2\sin 2x}{\cos 2x} + 4\right)$$

recall: $y = x^2 \cos 2x e^{4x}$

$$\therefore \frac{dy}{dx} =$$

$$= x^2 \cos 2x e^{4x}\left(\frac{2}{x} - \frac{2\sin 2x}{\cos 2x} + 4\right)$$

10) $y = \sin(3x^3 + 5)$

Let $u = 3x^3 + 5$

$$\frac{du}{dx} = 9x^2$$

$$y = \sin u$$

$$\frac{dy}{du} = \cos u$$

$$\therefore \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\therefore \frac{dy}{dx} = 9x^2 \cos u$$

$$= 9x^2 \cos(3x^3 + 5)$$