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MAT 102
Mechatronics Engineering
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1) A Particle moves along a Curve $x = 7t^2$, $y = 6t^2 - 4t$, $z = t - 5$, where t is the time find the velocity

Soln

The equation of the Particle moving along the curve is $x+y+z$

$$x + y + z = 7t^2 + (6t^2 - 4t) + (t - 5)$$
$$= 7t^2 + 6t^2 - 4t + t - 5$$
$$= 13t^2 - 3t - 5$$

Let $x + y + z = A$

$$A = 13t^2 - 3t - 5$$

$$\frac{dA}{dt} \text{ (velocity)} = 13 \times 2 \times t^{2-1} - 3 \times 1 \times t^{1-1} - 0$$

$$\frac{dA}{dt} = 26t - 3$$

velocity $= 26t - 3$

The value of "t" is not given in the question.

2) If $A = i + 4j - 4k$, $B = 2i - 3j + k$, $C = 4j - 3k$, find $A \times (B \times C)$

Solution

$A = i + 2j - 4k$, $B = 2i - 3j + k$, $C = 0i + 4j - 3k$

$(B \times C) \rightarrow$

i	$-j$	$-k$
2	-3	1
0	4	-3

$\Rightarrow i(-3 \times -3) - (1 \times 4) - j((2 \times -3) - (0 \times 1)) + k(2 \times 4 - (0 \times -3))$

$\Rightarrow i(9 - 4) - j(-6 - 0) + k(8 - 0)$

$5i + 6j + 8k$

ie $(B \times C) = 5i + 6j + 8k$

$\rightarrow A \times (B \times C) = \begin{array}{c|c|c} -i & -j & k \\ \hline 1 & 2 & -4 \\ 5 & 6 & 8 \end{array}$

$\rightarrow i(2\times 8 - (-4\times 6) - j(1\times 8 - (-4\times 5) + k(1\times 6 - (5\times 2)$

$\rightarrow i(16-(-24) - j(8-(-20) + k(6-10)$

$i(16+24) - j(8+20 + k(-4) = 40i - 28j - 4k$

Ans $= 40i - 28j - 4k$

3 Given $R = 4\sin 3t\, i + 4e^{3t} j + 7t^3 k$, find the integral of R with respect to t

Solution

$R = 4\sin 3t\, i + 4e^{3t} j + 7t^3 k$

$\int R = \int 4\sin 3t\, i + 4e^{3t} j + 7t^3 k$

$\int \sin x = -\cos x$

$\int e^{ax} = e^{ax}/x \rightarrow e^{ax}/a$

$\int R = 4(-\cos 3t\, i) + \frac{4e^{3t}}{3} j + \frac{7\times 3\times t^{3-1}}{3-1} k$

$= -4\cos 3t\, i + \frac{4e^{3t}}{3} j + \frac{21t^2}{2} k$

$= -4\cos 3t\, i + \frac{4e^{3t}}{3} j + \frac{21t^2}{2} k + c$

4) If $A = 7i + 2j - k$, $B = 2i + j + k$, $C = i + j + k$, find $(A+C)+(B-A)$

solution

$A = 7i + 2j - k$, $B = 2i + j + 4k$, $C = i + j + k$

$(A+C) - (B-A)$

$(A+C) \Rightarrow (7i + 2j - k) + (i + j + k)$

$\Rightarrow (7+1)i + (2+1)j + (-1+1)k$

$\Rightarrow 8i + 3j + 0k$ //

$(B-A) \Rightarrow (2i + j + 4k) - (7i + 2j - k)$

$(2-7)i + (1-2)j + (4-(-1)k$

$(-5)i + (-1)j + (4+1)k$

$\Rightarrow -5i - j + 5k$

$(A+C)\cdot(B-A) \Rightarrow (8i + 3j + 0k)\cdot(-5i - j + 5k)$

$\Rightarrow 8i(5i - j + 5k) + 3j(-5i - j + 5k) + 0k(-5i - j + 5k)$

$(8x-5)(i\cdot i) + (3x-1)(j\cdot j) + 0$

Note

$i\cdot i = k\cdot k = j\cdot j = 1$

$i\cdot j = i\cdot k = j\cdot k = 0$

$(8x-5)x1 + (3x-1)x1$

$-40 + (-3) \Rightarrow -40 - 3 = -43$ //

5 Find a unit vector tangent to the space curve $\Rightarrow x = t, y = t^2, z = t^3$ at point where $t = 1$

solution

$x = t, y = t^2, z = t^3$

The unit vector $\Rightarrow \frac{x}{|x+y+z|}, \frac{y}{|x+y+z|}, \frac{z}{|x+y+z|}$

$\Rightarrow |x+y+z| \Rightarrow \frac{t + t^2 + t^3}{\sqrt{t^2 + (t^2)^2 + (t^3)^2}}$, but $t = 1$

Modulus $= \sqrt{1^2 + (1^2)^2 + (1^3)^2} = \sqrt{1}$

Modulus $= 1$ //

The unit vector $= \frac{1}{1}, \frac{1^2}{1}, \frac{1^3}{1} = 1$

The unit vector tangent to the space curve $= \underline{1}$