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MBBS

Mat 101 Assignment

Examine whether or not these pair of lines are perpendicular to each other

1] $y - 3x - 2 = 0$ and $3y + x + 9 = 0$

2] $3y - 4 = 2x + 8$ and $y - 5 = x + 6$

3] Find the equations of the tangent and normal $x^2 + y^2 + 5xy - 11 = 0$ at $(1, 2)$

Answers

1] $y - 3x - 2 = 0$

rearrange into the form, $y = mx + c$

* $y = 3x + 2$; $\therefore m_1 = 3$

Also $3y + x + 9 = 0$

$\frac{3y}{3} = \frac{-x-9}{3}$

$y = \frac{-x}{3} - 3$; $m_2 = -\frac{1}{3}$

$\therefore m_1 m_2 = ?$

$3 \times -\frac{1}{3} = -1$ \therefore The lines are perpendicular ($m_1 m_2 = -1$)

2] $3y - 4 = 2x + 8$

rearrange into the form, $y = mx + c$

$\frac{3y}{3} = \frac{2x+12}{3}$, $m_1 = \frac{2}{3}$

Also $y - 5 = x + 6$

$y = x + 11$, $m_2 = 1$

$m_1 m_2 = ?$

$\frac{2}{3} \times 1 = \frac{2}{3}$ \therefore The lines are not perpendicular ($m_1 m_2 = \frac{2}{3}$)

$$3] x^2 + y^2 + 3xy - 11 = 0 \text{ at point } (1, 2)$$

$$\frac{dy}{dx} = 2x + 2y \frac{dy}{dx} + 3(x \frac{dy}{dx} + y) = 0$$

$$2x + 2y \frac{dy}{dx} + 3x \frac{dy}{dx} + 3y = 0$$

$$2y \frac{dy}{dx} + 3x \frac{dy}{dx} = -2x - 3y$$

$$\frac{dy}{dx} \frac{(2y+3x)}{(2y+3x)} = \frac{-2x-3y}{2y+3x}$$

$$\frac{dy}{dx} = \frac{-2x-3y}{2y+3x}$$

$$\therefore \frac{dy}{dx} \Big|_{x=1, y=2} \Rightarrow \frac{-2(1)-3(2)}{2(2)+3(1)} = \frac{-2-6}{4+3} = \underline{\underline{\frac{-8}{7}}}$$

$$y - y_1 = m_1(x - x_1)$$

$$y - 2 = \frac{-8}{7}(x - 1)$$

$$y - 2 = \frac{-8x}{7} + \frac{8}{7}$$

$$y - 2 = \frac{-8x + 8}{7}$$

$$7y - 14 = -8x + 8$$

$$7y + 8x - 14 - 8 = 0$$

$$7y + 8x - 22 = 0 \text{ eqn of tangent}$$

$$m_1 m_2 = -1$$

$$\frac{-8}{7} m_2 = -1$$

$$\therefore m_2 = \frac{7}{8}$$

$$y - y_1 = m_2(x - x_1)$$

$$y - 2 = \frac{7}{8}(x - 1)$$

$$y - 2 = \frac{7x}{8} - \frac{7}{8}$$

$$8y - 16 = 7x - 7$$

$$8y - 7x - 16 + 7 = 0$$

$$8y - 7x - 9 = 0 \text{ eqn of the normal}$$