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19/MHS11/070. Pharmacy

MAT104

(1) $y = \frac{1}{x-2}$

- The function is defined for all real numbers except $x = 2$.
- The domain is the set of real numbers except $x = 2$.
- The codomain is the set of real numbers except $y = 0$

(2) $K = \ln V$

$\frac{dK}{dV} = \frac{1}{V}$

(3) $\cancel{3}2x - 3y - 2 = 0$

$-3y = 2 - 2x$

$y = \frac{2-2x}{-3}$

$y = \frac{2x+2}{3}$; $\frac{2(x+1)}{3}$

(b) $x^2 + y^2 = 4$

$y^2 = 4 - x^2$

$y = \pm\sqrt{4 - x^2}$

(4) find dp/dt, $p = \sin^{-1}t$

$p = \frac{t}{\sin}$; $t = \sin p$

$\frac{dt}{dp} = \cos p$; $\frac{dp}{dt} = \frac{1}{\cos p}$

Recall, $\cos^2 y + \sin^2 y = 1$

$\cos y = \pm\sqrt{1 - \sin^2 y}$

$t = \sin p$

$\therefore \cos p = \sqrt{1 - t^2}$

Hence, $dp/dt = 1/\sqrt{(1-t^2)}$

(5) $f(x) = 2x^2 - 5$; $g(x) = 4x - 2$

$f\circ g(x) = 2(4x-2)^2 - 5$

$= 2(16x^2 - 16x + 4) - 5$

$= 32x^2 - 32x + 8 - 5$

$= 32x^2 - 32x + 3$

$g\circ f(x) = 4(2x^2 - 5) - 2$

$= 8x^2 - 20 - 2$

$= 8x^2 - 22.$

(6) Show that $f(x) = f_e(x) + f_o(x)$

$f(x) = 3x^2 - 2x + 1$

$f_e(x) = \frac{f(x) + f(-x)}{2}$

$f(-x) = \frac{3x^2 - 2x + 1 + (3x^2 + 2x + 1)}{2}$

$= \frac{6x^2 + 2}{2} = 3x^2 + 1$

$f_o(x) = \frac{3x^2 - 2x + 1 - (3x^2 + 2x + 1)}{2}$

$= \frac{-4x}{2} = -2x$

$f_e(x) + f_o(x) = 3x^2 + 1 - 2x$

$= 3x^2 - 2x + 1$

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(7) Differentiate $y = \cos x$

$y + dy = \cos(x + dx)$

$y = \cos x$

$dy = \cos(x + dx) - \cos x$ (1)

Recall (2)

$\cos(A+B) - \cos(A-B) = -2\sin A \sin B$

comparing (1) & (2)

$A + B = x + dx$ — (3)

$A - B = x$ — (4)

Adding (3) & (4) and subtracting (3) & (4)

$2A = 2x + dx \quad B = \frac{dx}{2}$

$A = \frac{2x + dx}{2}$

$A = x + \frac{dx}{2}$

Comparing (1) & (2)

$dy = \cos(x + dx) - \cos x$

$= 2\sin(x + dx/2)\sin(dx/2)$

Dividing through by dx

$\frac{dy}{dx} = \frac{-2\sin(x + dx/2)\sin(dx/2)}{dx}$

$\frac{dy}{dx} = -\sin(x + \frac{dx}{2})\sin(dx/2)$

$= -\sin(x + dx/2) \times \frac{\sin(dx/2)}{dx/2}$

Taking limit $dx \to 0$

$\lim \frac{\sin dx/2}{dx/2} = 1$

$\frac{dy}{dx} = -\sin(x + 0/2) \times 1$

$\lim_{dx \to 0}$

$\frac{dy}{dx} = -\sin x$

(8) $y = 3t^2$; $x = 1/t^2$

$\frac{dy}{dx} = \frac{dy}{dx} \times \frac{dt}{dx}$

$= \frac{dy}{dx} \div \frac{dx}{dt}$

$\frac{dy}{dt} = 6t$; $\frac{dx}{dt} = \frac{-2}{t^3}$

$\frac{dy}{dx} = 6t \div \frac{-2}{t^3}$

$= \frac{\cancel{6t} \times -2}{\cancel{t^3}} = \frac{-6 \times -2}{t^2} = \frac{12}{t^2}$

$= \frac{12}{t^2}$

(9) $y = x^2 \cos 2x \; e^{4x}$

taking $\log_e$ of both sides.

$\ln y = \ln x^2 + \ln\cos 2x + \ln e^{4x}$

Differentiating both with x

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x^2}(2x) + \frac{1}{\cos 2x}(-2\sin 2x)$

$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x} - \frac{2\sin 2x}{\cos 2x} + 4$

Multiplying both sides by y

$\frac{dy}{dx} = y\left(\frac{2}{x} - \frac{2\sin 2x}{\cos 2x} + 4\right)$

$= x^2 \cos 2x \, e^{4x} \times \frac{2}{x} - \frac{2\sin x}{\cos x} + 4$

(10) $y = \sin(3x^3+5)$

$u = 3x^3+5$

$y = \sin u$

$\frac{dy}{du} = \cos u$

$\frac{du}{dx} = 9x^2$

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$\frac{dy}{dx} = \cos u \times 9x^2$

$\frac{dy}{dx} = 9x^2 \cos u$

$\therefore \frac{dy}{dx} = 9x^2 \cos 3x^3+5$

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