MAT 104 ASSIGNMENT

NAME : TIAMIYU AISHA BOLAJI
DEPARTMENT : PHARMACY
MATRIC NO : 19/MHS11/136

1. For what values of x is the function $y = \frac{1}{x-2}$ defined? state the domain & co-domain.

SOLUTION

(a) The function is defined for all real numbers except $x = 2$.

(b) Domain $\Rightarrow$ All real numbers except 2.

Co-domain $\Rightarrow$ All real numbers except 0.

2. If $K = \ln V$, differentiate k.

$$\frac{dk}{dx}(\ln V) = \frac{1}{V}$$

3. Express y as an explicit function of x in the following. (a) $2x - 3y - 2 = 0$. (b) $x^2 + y^2 = 4$.

SOLUTION

(a)
$$2x - 3y - 2 = 0$$
$$2x - 3y = 2$$
$$-3y = 2 - 2x$$
$$y = \frac{-2 - 2x}{3}$$

(b)
$$x^2 + y^2 = 4$$
$$y^2 = 4 - x^2$$
$$y = \pm\sqrt{4 - x^2}$$

4) If $p = \sin^{-1}t$, find the derivative of p.

SOLUTION

$$\frac{dp}{dx}(\sin^{-1}t) = \frac{1}{\sqrt{1-t^2}}$$

5) If $f(x) = 2x^2 - 5$ and $g(x) = 4x - 2$, find $f \circ g(x)$ and $g \circ f(x)$.

SOLUTION

(a)
$$f \circ g(x) = 2(4x-2)^2 - 5$$
$$= 2(4x-2)(4x-2) - 5$$
$$f \circ g(x) = 2(16x^2 - 16x + 4) - 5$$
$$= 32x^2 - 32x + 8 - 5$$
$$= 32x^2 - 32x + 3$$

(b)
$$g \circ f(x) = 4(2x^2 - 5) - 2$$
$$= 8x^2 - 20 - 2$$
$$= 8x^2 - 18$$

6) If $f(x) = 3x^2 - 2x + 1 = 0$, Show that $f_e(x) + f_o(x) = f(x)$.

SOLUTION

$$f(-x) = 3(-x)^2 - 2(-x) + 1$$
$$= 3x^2 + 2x + 1$$
$$f_e(x) = \frac{3x^2 - 2x + 1 + 3x^2 + 2x + 1}{2}$$
$$f_e(x) = \frac{6x^2 + 2}{2} = \frac{\cancel{2}(3x^2+1)}{\cancel{2}}$$
$$f_e(x) = 3x^2 + 1$$
$$f_o(x) = \frac{f(x) - f(-x)}{2}$$
$$f_o(x) = \frac{3x^2 - 2x + 1 - 3x^2 + 2x + 1}{2}$$

$f_o(x) = \frac{3x^2-2x+1-3x^2-2x-1}{2}$

$f_o(x) = -\frac{\cancel{4}^{2}x}{\cancel{2}_{1}} = -2x$

$f(x) = 3x^2+1+(-2x) = 3x^2-2x+1$

Hence, $f(x) = f_e(x) + f_o(x)$.

7. Differentiate $y = \cos x$ from first principles.

SOLUTION

Let $h = \delta x$

$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$\lim_{h\to 0} \frac{\cos(x+h)-\cos(x)}{h}$

using $\cos(a+b) = \cos a\cos b - \sin a \sin b$

$= \lim_{h\to 0} \frac{(\cos x\cosh - \sin x \sinh)\cos x}{h}$

$\lim_{h\to 0} \frac{\cos x(\cosh -1) - \sin x\sinh}{h}$

$\lim_{h\to 0} \frac{\cos x(\cosh -1)}{h} - \frac{\sin x \sinh}{h}$

$\lim_{h\to 0} \cos x\left(\frac{\cosh -1}{h}\right) - \sin x\left(\frac{\sinh}{h}\right)$

$\lim_{h\to 0} \cos x(0) - \sin x(1)$

$\therefore \frac{d}{dx}(\cos x) = -\sin x$

8. Find $\frac{dy}{dx}$, If $y = 3t^2$ and $x = \frac{1}{t^2}$

SOLUTION

$y = 3t^2, \quad \frac{dy}{dt} = 6t$

$x = \frac{1}{t^2}, \quad \frac{dx}{dt} = -2t^{-3} = \frac{-2}{t^3}$

$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = \frac{6t}{\frac{-2}{t^3}}$

$= 6t \times \frac{-2}{t^3} = \frac{12\cancel{t}}{\cancel{t}^3}$

$\therefore \frac{dy}{dx} = \frac{12}{t^2}$

9) Find $\frac{dy}{dx}$ if $y = x^2\cos 2x e^{4x}$

SOLUTION

Taking $\log_e$ of both sides.

$\ln y = \ln x^2 + \ln\cos 2x + \ln e^{4x}$

Differentiating both sides wrt x

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x^2}(2x) + \frac{1(-2\sin 2x)}{\cos 2x} + 4$

$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x} - \frac{2\sin 2x}{\cos 2x} + 4$

multiplying both sides by y:

$\frac{dy}{dx} = y\left(\frac{2}{x} - \frac{2\sin 2x}{\cos 2x} + 4\right)$

$\frac{dy}{dx} = x^2\cos 2x e^{4x} \times \left(\frac{2}{x} - \frac{2\sin x}{\cos x} + 4\right)$

10) Given that $y = \sin(3x^3+5)$. Find the derivative of y.

SOLUTION

$y = \sin(3x^3+5)$

Let $U = 3x^3+5$; $\frac{du}{dx} = 9x^2$

$\Rightarrow y = \sin U : \frac{dy}{du} = \cos U$

$\therefore \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = \cos U \times 9x^2$

$\frac{dy}{dx} = 9x^2 \cos U$

but $U = 3x^3+5$

$\therefore \frac{dy}{dx} = 9x^2\cos 3x^3+5.$